The Riemann Zeta Function Theory And Applications Aleksandar Ivic

The Riemann zeta function was introduced by L. Euler (1737) in connection with questions about the distribution of prime numbers. Later, B. Riemann (1859) derived deeper results about the prime numbers by considering the zeta function in the complex variable. The famous Riemann Hypothesis, asserting that all of the non-trivial zeros of zeta are on a critical line in the complex plane, is one of the most important unsolved problems in modern mathematics. The present book consists of two parts. The first part covers classical material about the zeros of the Riemann zeta function with applications to the distribution of prime numbers, including those made by Riemann himself, F. Carlson, and Hardy-Littlewood. The second part gives a complete presentation of Levinson's method for zeros on the critical line, which allows one to prove, in particular, that more than one-third of non-trivial zeros of zeta are on the critical line. This approach and some results concerning integrals of Dirichlet polynomials are new. There are also technical lemmas which can be useful in a broader context.

A graduate-level account of an important recent result concerning the Riemann zeta function.

This text covers exponential integrals and sums, 4th power moment, zero-free region, mean value estimates over short intervals, higher power moments, omega results, zeros on the critical line, zero-density estimates, and more. 1985 edition.

Exploring the Riemann Zeta Function: 190 years from Riemann's Birth presents a collection of chapters contributed by eminent experts devoted to the Riemann Zeta Function, its generalizations, and their various applications to several scientific disciplines, including Analytic Number Theory, Harmonic Analysis, Complex Analysis, Probability Theory, and related subjects. The book focuses on both old and new results towards the solution of long-standing problems as well as it features some key historical remarks. The purpose of this volume is to present in a unified way broad and deep areas of research in a self-contained manner. It will be particularly useful for graduate courses and seminars as well as it will make an excellent reference tool for graduate students and researchers in Mathematics, Mathematical Physics, Engineering and Cryptography.

This book examines the application of complex analysis methods to the theory of prime numbers. In an easy to understand manner, a connection is established between arithmetic problems and those of zero distribution for special functions. Main achievements in this field of mathematics are described. Indicated is a connection between the famous Riemann zeta-function and the structure of the universe, information theory, and quantum mechanics. The theory of Riemann zeta-function and, specifically, distribution of its zeros are presented in a concise and comprehensive way. The full proofs of some modern theorems are given. Significant methods of the analysis are also demonstrated as applied to fundamental problems of number theory.

The Theory of the Riemann Zeta-functionOxford University Press

The Riemann Hypothesis has become the Holy Grail of mathematics in the century and a half since 1859 when Bernhard Riemann, one of the extraordinary mathematical talents of the 19th century, originally posed the problem. While the problem is notoriously difficult, and complicated even to state carefully, it can be loosely formulated as "the number of integers with an even number of prime factors is the same as the number of integers with an odd number of prime factors." The Hypothesis makes a very precise connection between two seemingly unrelated mathematical objects, namely prime numbers and the zeros of analytic functions. If solved, it would give us profound insight into number theory and, in particular, the nature of prime numbers. This book is an introduction to the theory surrounding the Riemann Hypothesis. Part I serves as a compendium of known results and as a primer for the material presented in the 20 original papers contained in Part II. The original papers place the material into historical context and illustrate the motivations for research on and around the Riemann Hypothesis. Several of these papers focus on computation of the zeta function, while others give proofs of the Prime Number Theorem, since the Prime Number Theorem is so closely connected to the Riemann Hypothesis. The text is suitable for a graduate course or seminar or simply as a reference for anyone interested in this extraordinary conjecture.

An introduction to the analytic techniques used in the investigation of zeta functions through the example of the Riemann zeta function. It emphasizes central ideas of broad application, avoiding technical results and the customary function-theoretic appro

Two major subjects are treated in this book. The main one is the theory of Bernoulli numbers and the other is the theory of zeta functions. Historically, Bernoulli numbers were introduced to give formulas for the sums of powers of consecutive integers. The real reason that they are indispensable for number theory, however, lies in the fact that special values of the Riemann zeta function can be written by using Bernoulli numbers. This leads to more advanced topics, a number of which are treated in this book: Historical remarks on Bernoulli numbers and the formula for the sum of powers of consecutive integers; a formula for Bernoulli numbers by Stirling numbers; the Clausen–von Staudt theorem on the denominators of Bernoulli numbers; Kummer's congruence between Bernoulli numbers and a related theory of p-adic measures; the Euler–Maclaurin summation formula; the functional equation of the Riemann zeta function and the Dirichlet L functions, and their special values at suitable integers; various formulas of exponential sums expressed by generalized Bernoulli numbers; the relation between ideal classes of orders of quadratic fields and equivalence classes of binary quadratic forms; class number formula for positive definite binary quadratic forms; congruences between some class numbers and Bernoulli numbers; simple zeta functions of prehomogeneous vector spaces; Hurwitz numbers; Barnes multiple zeta functions and their special values; the functional equation of the double zeta functions; and poly-Bernoulli numbers. An appendix by Don Zagier on curious and exotic identities for Bernoulli numbers is also supplied. This book will be enjoyable both for amateurs and for professional researchers. Because the logical relations between the chapters are loosely connected, readers can start with any chapter depending on their interests. The expositions of the topics are not always typical, and some parts are completely new.

The aim of the series is to present new and important developments in pure and applied mathematics. Well established in the community over two decades, it offers a large library of mathematics including several important classics. The volumes supply thorough and detailed expositions of the methods and ideas essential to the topics in question. In addition, they convey their relationships to other parts of mathematics. The series is addressed to advanced readers wishing to thoroughly study the topic. Editorial Board Lev Birbrair, Universidade Federal do Ceará, Fortaleza, Brasil Victor P. Maslov, Russian Academy of Sciences, Moscow, Russia Walter D. Neumann, Columbia University, New York, USA Markus J. Pflaum, University of Colorado, Boulder, USA Dierk Schleicher, Jacobs University, Bremen, Germany

The subject of this book is probabilistic number theory. In a wide sense probabilistic number theory is part of the analytic number theory, where the methods and ideas of probability theory are used to study the distribution of values of arithmetic objects. This is usually complicated, as it is difficult to say anything about their concrete values. This is why the following problem is usually investigated: given some set, how often do values of an arithmetic object get into this set? It turns out that this frequency follows strict mathematical laws. Here we discover an analogy with quantum mechanics where it is impossible to describe the chaotic behaviour of one particle, but that large numbers of particles obey statistical laws. The objects of investigation of this book are Dirichlet series, and, as the title shows, the main attention is devoted to the Riemann zeta-function. In studying the distribution of values of Dirichlet series the weak convergence of probability measures on different spaces (one of the principle asymptotic probability theory methods) is used. The application of this method was launched by H. Bohr in the third decade of this century and it was implemented in his works together with B. Jessen. Further development of this idea was made in the papers of B. Jessen and A. Wintner, V. Borchsenius and B.

Zeta and q-Zeta Functions and Associated Series and Integrals is a thoroughly revised, enlarged and updated version of Series Associated with the Zeta and Related Functions. Many of the chapters and sections of the book have been significantly modified or rewritten, and a new chapter on the theory and applications of the basic (or q-) extensions of various special functions is included. This book will be invaluable because it covers not only detailed and systematic presentations of the theory and applications of the various methods and techniques used in dealing with many different classes of series and integrals associated with the Zeta and related functions, but stimulating historical accounts of a large number of problems and well-classified tables of series and integrals. Detailed and systematic presentations of the theory and applications of the various methods and techniques used in dealing with many different classes of series and integrals associated with the Zeta and related functions

A fractal drum is a bounded open subset of R. m with a fractal boundary. A difficult problem is to describe the relationship between the shape (geo metry) of the drum and its sound (its spectrum). In this book, we restrict ourselves to the one-dimensional case of fractal strings, and their higher dimensional analogues, fractal sprays. We develop a theory of complex di mensions of a fractal string, and we study how these complex dimensions relate the geometry with the spectrum of the fractal string. We refer the reader to [Berrl-2, Lapl-4, LapPol-3, LapMal-2, HeLapl-2] and the ref erences therein for further physical and mathematical motivations of this work. (Also see, in particular, Sections 7. 1, 10. 3 and 10. 4, along with Ap pendix B.) In Chapter 1, we introduce the basic object of our research, fractal strings (see [Lapl-3, LapPol-3, LapMal-2, HeLapl-2]). A 'standard fractal string' is a bounded open subset of the real line. Such a set is a disjoint union of open intervals, the lengths of which form a sequence which we assume to be infinite. Important information about the geometry of . c is contained in its geometric zeta function (c(8) = L lj. j=l 2 Introduction We assume throughout that this function has a suitable meromorphic ex tension. The central notion of this book, the complex dimensions of a fractal string . c, is defined as the poles of the meromorphic extension of (c.

Monograph on most important topic in number theory.

This book provides both classical and new results in Reimann Zeta-Function theory, one of the most important problems in analytic number theory. These results have application in solving problems in multiplicative number theory, such as power moments, the zero-free region, and the zero density estimates. The book also furnishes annotated proofs, end-of-chapter notes, historical discussions and references.

In this thesis we provide a body of knowledge that concerns Riemann zeta-function and its generalizations in a cohesive manner. In particular, we have studied and mentioned some recent results regarding Hurwitz and Lerch functions, as well as Dirichlet's L-function. We have also investigated some fundamental concepts related to these functions and their universality properties. In addition, we also discuss different formulations and approaches to the proof of the Prime Number Theorem and the Riemann Hypothesis. These two topics constitute the main theme of this thesis. For the Prime Number Theorem, we provide a thorough discussion that compares and contrasts Norbert Wiener's proof with that of Newman's short proof. We have also related them to Hadamard's and de la Vallee Poussin's original proofs written in 1896. As far as the Riemann Hypothesis is concerned, we discuss some recent results related to equivalent formulations of the Riemann Hypothesis as well as the Generalized Riemann Hypothesis. keywords: Riemann zeta function, Hurwtiz zeta function, L-functions, Dedekind zeta function, Universality, Prime Number Theorem, Riemann Hypothesis, Generalized Riemann Hypothesis, Analytic Number Theory, Special Functions.

This volume provides a systematic survey of almost all the equivalent assertions to the functional equations - zeta symmetry - which zeta-functions satisfy, thus streamlining previously published results on zeta-functions. The equivalent relations are given in the form of modular relations in Fox H-function series, which at present include all that have been considered as candidates for ingredients of a series. The results are presented in a clear and simple manner for readers to readily apply without much knowledge of zeta-functions. This volume aims to keep a record of the 150-year-old heritage starting from Riemann on zeta-functions, which are ubiquitous in all mathematical sciences, wherever there is a notion of the norm. It provides almost all possible equivalent relations to the zeta-functions without requiring a reader's deep knowledge on their definitions. This can be an ideal reference book for those studying zeta-functions.

This volume contains the proceedings of the conference "Casimir Force, Casimir Operators and the Riemann Hypothesis – Mathematics for Innovation in Industry and Science" held in November 2009 in Fukuoka (Japan). The conference focused on the following topics: Casimir operators in harmonic analysis and representation theory Number theory, in particular zeta functions and cryptography Casimir force in physics and its relation with nano-science Mathematical biology Importance of mathematics for innovation in industry

The Riemann zeta function is one of the most studied objects in mathematics, and is of fundamental importance. In this book, based on his own research, Professor Motohashi shows that the function is closely bound with automorphic forms and that many results from there can be woven with techniques and ideas from analytic number theory to yield new insights into, and views of, the zeta function itself. The story starts with an elementary but unabridged treatment of the spectral resolution of the non-Euclidean Laplacian and the trace formulas. This is achieved by the use of standard tools from analysis rather than any heavy machinery, forging a substantial aid for beginners in spectral theory as well. These ideas are then utilized to unveil an image of the zeta-function, first perceived by the author, revealing it to be the main gem of a necklace composed of all automorphic L-functions. In this book, readers will find a detailed account of one of the most fascinating stories in the development of number theory, namely the fusion of two main fields in mathematics that were previously studied separately.

Superb high-level study of one of the most influential classics in mathematics examines landmark 1859 publication entitled “On the Number of Primes Less Than a Given Magnitude,” and traces developments in theory inspired by it. Topics include Riemann's main formula, the prime number theorem, the Riemann-Siegel formula, large-scale computations, Fourier analysis, and other related topics. English translation of Riemann's original document appears in the Appendix.

This is the first introductory book on multiple zeta functions and multiple polylogarithms which are the generalizations of the Riemann zeta function and the classical polylogarithms, respectively, to the multiple variable setting. It contains all the basic concepts and the important properties of these functions and their special values. This book is aimed at graduate students, mathematicians and physicists who are interested in this current active area of research. The book will provide a detailed and comprehensive introduction to these objects, their fascinating properties and interesting relations to other mathematical subjects, and various generalizations such as their q-analogs and their finite versions (by taking partial sums modulo suitable prime powers). Historical notes and exercises are provided at the end of each chapter. Contents:Multiple Zeta FunctionsMultiple Polylogarithms (MPLs)Multiple Zeta Values (MZVs)Drinfeld Associator and Single-Valued MZVsMultiple Zeta Value IdentitiesSymmetrized Multiple Zeta Values (SMZVs)Multiple Harmonic Sums (MHSs) and Alternating VersionFinite Multiple Zeta Values and Finite Euler Sumsq-Analogs of Multiple Harmonic (Star) Sums Readership: Advanced undergraduates and graduate students in mathematics, mathematicians interested in multiple zeta values. Key Features:For the first time, a detailed explanation of the theory of multiple zeta values is given in book form along with numerous illustrations in explicit examplesThe book provides for the first time a comprehensive introduction to multiple polylogarithms and their special values at roots of unity, from the basic definitions to the more advanced topics in current active researchThe book contains a few quite intriguing results relating the special values of multiple zeta functions and multiple polylogarithms to other branches of mathematics and physics, such as knot theory and the theory of motivesMany exercises contain supplementary materials which deepens the reader's understanding of the main text

The Riemann zeta-function is our most important tool in the study of prime numbers, and yet the famous "Riemann hypothesis" at its core remains unsolved. This book studies the theory from every angle and includes new material on recent work.

Studying the relationship between the geometry, arithmetic and spectra of fractals has been a subject of significant interest in contemporary mathematics. This book contributes to the literature on the subject in several different and new ways. In particular, the authors provide a rigorous and detailed study of the spectral operator, a map that sends the geometry of fractal strings onto their spectrum. To that effect, they use and develop methods from fractal geometry, functional analysis, complex analysis, operator theory, partial differential equations, analytic number theory and mathematical physics.Originally, M L Lapidus and M van Frankenhuijsen 'heuristically' introduced the spectral operator in their development of the theory of fractal strings and their complex dimensions, specifically in their reinterpretation of the earlier work of M L Lapidus and H Maier on inverse spectral problems for fractal strings and the Riemann hypothesis.One of the main themes of the book is to provide a rigorous framework within which the corresponding question 'Can one hear the shape of a fractal string?' or, equivalently, 'Can one obtain information about the geometry of a fractal string, given its spectrum?' can be further reformulated in terms of the invertibility or the quasi-invertibility of the spectral operator.The infinitesimal shift of the real line is first precisely defined as a differentiation operator on a family of suitably weighted Hilbert spaces of functions on the real line and indexed by a dimensional parameter c. Then, the spectral operator is defined via the functional calculus as a function of the infinitesimal shift. In this manner, it is viewed as a natural 'quantum' analog of the Riemann zeta function. More precisely, within this framework, the spectral operator is defined as the composite map of the Riemann zeta function with the infinitesimal shift, viewed as an unbounded normal operator acting on the above Hilbert space.It is shown that the quasi-invertibility of the spectral operator is intimately connected to the existence of critical zeros of the Riemann zeta function, leading to a new spectral and operator-theoretic reformulation of the Riemann hypothesis. Accordingly, the spectral operator is quasi-invertible for all values of the dimensional parameter c in the critical interval (0,1) (other than in the midfractal case when c =1/2) if and only if the Riemann hypothesis (RH) is true. A related, but seemingly quite different, reformulation of RH, due to the second author and referred to as an 'asymmetric criterion for RH', is also discussed in some detail: namely, the spectral operator is invertible for all values of c in the left-critical interval (0,1/2) if and only if RH is true.These spectral reformulations of RH also led to the discovery of several 'mathematical phase transitions' in this context, for the shape of the spectrum, the invertibility, the boundedness or the unboundedness of the spectral operator, and occurring either in the midfractal case or in the most fractal case when the underlying fractal dimension is equal to ½ or 1, respectively. In particular, the midfractal dimension c=1/2 is playing the role of a critical parameter in quantum statistical physics and the theory of phase transitions and critical phenomena.Furthermore, the authors provide a 'quantum analog' of Voronin's classical theorem about the universality of the Riemann zeta function. Moreover, they obtain and study quantized counterparts of the Dirichlet series and of the Euler product for the Riemann zeta function, which are shown to converge (in a suitable sense) even inside the critical strip.For pedagogical reasons, most of the book is devoted to the study of the quantized Riemann zeta function. However, the results obtained in this monograph are expected to lead to a quantization of most classic arithmetic zeta functions, hence, further 'naturally quantizing' various aspects of analytic number theory and arithmetic geometry.The book should be accessible to experts and non-experts alike, including mathematics and physics graduate students and postdoctoral researchers, interested in fractal geometry, number theory, operator theory and functional analysis, differential equations, complex analysis, spectral theory, as well as mathematical and theoretical physics. Whenever necessary, suitable background about the different subjects involved is provided and the new work is placed in its proper historical context. Several appendices supplementing the main text are also included.

In this text, the famous zeros of the Riemann zeta function and its generalizations (L-functions, Dedekind and Selberg zeta functions)are analyzed through several zeta functions built over those zeros.

Zeta functions have been a powerful tool in mathematics over the last two centuries. This book considers a new class of non-commutative zeta functions which encode the structure of the subgroup lattice in infinite groups. The book explores the analytic behaviour of these functions together with an investigation of functional equations. Many important examples of zeta functions are calculated and recorded providing an important data base of explicit examples and methods for calculation.

Zeta-function regularization is a powerful method in perturbation theory, and this book is a comprehensive guide for the student of this subject. Everything is explained in detail, in particular the mathematical difficulties and tricky points, and several applications are given to show how the procedure works in practice, for example in the Casimir effect, gravity and string theory, high-temperature phase transition, topological symmetry breaking, and non-commutative spacetime. The formulae, some of which are new, can be directly applied in creating physically meaningful, accurate numerical calculations. The book acts both as a basic introduction and a collection of exercises for those who want to apply this regularization procedure in practice. Thoroughly revised, updated and expanded, this new edition includes novel, explicit formulas on the general quadratic, the Chowla-Selberg series case, an interplay with the Hadamard calculus, and also features a fresh chapter on recent cosmological applications, including the calculation of the vacuum energy fluctuations at large scale in braneworld and other models.

Two major subjects are treated in this book. The main one is the theory of Bernoulli numbers and the other is the theory of zeta functions. Historically, Bernoulli numbers were introduced to give formulas for the sums of powers of consecutive integers. The real reason that they are indispensable for number theory, however, lies in the fact that special values of the Riemann zeta function can be written by using Bernoulli numbers. This leads to more advanced topics, a number of which are treated in this book: Historical remarks on Bernoulli numbers and the formula for the sum of powers of consecutive integers; a formula for Bernoulli numbers by Stirling numbers; the Clausen–von Staudt theorem on the denominators of Bernoulli numbers; Kummer's congruence between Bernoulli numbers and a related theory of p-adic measures; the Euler–Maclaurin summation formula; the functional equation of the Riemann zeta function and the Dirichlet L functions, and their special values at suitable integers; various formulas of exponential sums expressed by generalized Bernoulli numbers; the relation between ideal classes of orders of quadratic fields and equivalence classes of binary quadratic forms; class number formula for positive definite binary quadratic forms; congruences between some class numbers and Bernoulli numbers; simple zeta functions of prehomogeneous vector spaces; Hurwitz numbers; Barnes multiple zeta functions and their special values; the functional equation of the doub le zeta functions; and poly-Bernoulli numbers. An appendix by Don Zagier on curious and exotic identities for Bernoulli numbers is also supplied. This book will be enjoyable both for amateurs and for professional researchers. Because the logical relations between the chapters are loosely connected, readers can start with any chapter depending on their interests. The expositions of the topics are not always typical, and some parts are completely new.

This is a modern introduction to the analytic techniques used in the investigation of zeta functions, through the example of the Riemann zeta function. Riemann introduced this function in connection with his study of prime numbers and from this has developed the subject of analytic number theory. Since then many other classes of 'zeta function' have been introduced and they are now some of the most intensively studied objects in number theory. Professor Patterson has emphasised central ideas of broad application, avoiding technical results and the customary function-theoretic approach. Thus, graduate students and non-specialists will find this an up-to-date and accessible introduction, especially for the purposes of algebraic number theory. There are many exercises included throughout, designed to encourage active learning.

This is an advanced text on the Riemann zeta-function, a continuation of theauthor's earlier book. It presents the most recent results on mean values, many of which had not yet appeared in print at the time of the writing of the text. An especially detailed discussion is given of the second and the fourth moment, and the latter is studied by the use of spectral theory, one of the most powerful methods used lately in analytic number theory. The book presupposes a reasonable knowledge of zeta-function theory and complex analysis. It will be of great use to the researchers in the field, an to all those who wish to get well acquainted with the subject or who have the need for application of zeta-function theory.