Minimax Methods In Critical Point Theory With Applications To Differential Equations Cbms Regional Conference Series In Mathematics

Many boundary value problems are equivalent to Au=O (1) where A : X --+ Y is a mapping between two Banach spaces. When the problem is variational, there exists a differentiable functional rand inf.

This text starts at the foundations of the field, and is accessible with some background in functional analysis. As such, the book is ideal for classroom of self study. The new material covered also makes this book a must read for researchers in the theory of critical points.

This paper contains a slightly expanded version of the series of lectures given by the author at the Third International Symposium on Differential Equations and Differential Geometry held in Changchun, China during parts of August and September of 1982. The lectures describe some of the results obtained using minimax methods in critical point theory during the past several years and give applications of the abstract results to differential equations. In particular existence theorems are obtained for many boundary value problems for semilinear elliptic partial differential equations.

FACHGEB The last decade has seen a tremendous development in critical point theory in infinite dimensional spaces and its application to nonlinear boundary value problems. In particular, striking results were obtained in the classical problem of periodic solutions of Hamiltonian systems. This book provides a systematic presentation of the most basic tools of critical point theory: minimization, convex functions and Fenchel transform, dual least action principle, Ekeland variational principle, minimax methods, Lusternik- Schirelmann theory for Z2 and S1 symmetries, Morse theory for possibly degenerate critical points and non-degenerate critical manifolds. Each technique is illustrated by applications to the discussion of the existence, multiplicity, and bifurcation of the periodic solutions of Hamiltonian systems. Among the treated questions are the periodic solutions with fixed period or fixed energy of autonomous systems, the existence of subharmonics in the non-autonomous case, the asymptotically linear Hamiltonian systems, free and forced superlinear problems. Application of those results to the equations of mechanical pendulum, to Josephson systems of solid state physics and to questions from celestial mechanics are given. The aim of the book is to introduce a reader familiar to more classical techniques of ordinary differential equations to the powerful approach of modern critical point theory. The style of the exposition has been adapted to this goal. The new topological tools are introduced in a progressive but detailed way and immediately applied to differential equation problems. The abstract tools can also be applied to partial differential equations and the reader will also find the basic references in this direction in the bibliography of more than 500 items which concludes the book. ERSCHEIN

This volume takes up various topics in Mathematical Analysis including boundary and initial value problems for Partial Differential Equations and Functional Analytic methods.Topics include linear elliptic systems for composite material ? the coefficients may jump from domain to domain; Stochastic Analysis ? many applied problems involve evolution equations with random terms, leading to the use of stochastic analysis.The proceedings have been selected for coverage in: ? Index to Scientific & Technical Proceedings (ISTP CDROM version / ISI Proceedings)? CC Proceedings ? Engineering & Physical Sciences

The book provides an introduction to minimax methods in critical point theory and shows their use in existence questions for nonlinear differential equations. An expanded version of the author's 1984 CBMS lectures, this volume is the first monograph devoted solely to these topics. Among the abstract questions considered are the following: the mountain pass and saddle point theorems, multiple critical points for functionals invariant under a group of symmetries, perturbations from symmetry, and variational methods in bifurcation theory. The book requires some background in functional analysis and differential equations, especially elliptic partial differential equations. It is addressed to mathematicians interested in differential equations and/or nonlinear functional analysis, particularly critical point theory.

This self-contained monograph presents methods for the investigation of nonlinear variational problems. These methods are based on geometric and topological ideas such as topological index, degree of a mapping, Morse-Conley index, Euler characteristics, deformation invariant, homotopic invariant, and the Lusternik-Shnirelman category. Attention is also given to applications in optimisation, mathematical physics, control, and numerical methods. Audience: This volume will be of interest to specialists in functional analysis and its applications, and can also be recommended as a text for graduate and postgraduate-level courses in these fields.

This paper contains the written of a series of lectures presented by the author at the American Mathematical Society Summer Institute on Nonlinear Functional Analysis and Nonlinear Differential Equations. These lectures are an introduction to minimax techniques for finding critical points of functionals, especially functionals possessing symmetries. Applications are made to semilinear elliptic partial differential equations and Hamiltonian systems of ordinary differential equations. (Author).

This book will be especially useful for post-graduate students and researchers interested in the fixed point theory, particularly in topological methods in nonlinear analysis, differential equations and dynamical systems. The content is also likely to stimulate the interest of mathematical economists, population dynamics experts as well as theoretical physicists exploring the topological dynamics.

Minimax Methods in Critical Point Theory with Applications to Differential EquationsAmerican Mathematical Soc.

As is well known, The Great Divide (a.k.a. The Continental Divide) is formed by the Rocky Mountains stretching from north to south across North America. It creates a virtual "stone wall" so high that wind, rain, snow, etc. cannot cross it. This keeps the weather distinct on both sides. Since railroad trains cannot climb steep grades and tunnels through these mountains are almost formidable, the Canadian Pacific Railroad searched for a mountain pass providing the lowest grade for its tracks. Employees discovered a suitable mountain pass, called the Kicking Horse Pass, el. 5404 ft., near Banff, Alberta. (One can speculate as to the reason for the name.) This pass is also used by the Trans-Canada Highway. At the highest point of the pass the railroad tracks are horizontal with mountains rising on both sides. A mountain stream divides into two branches, one flowing into the Atlantic Ocean and the other into the Pacific. One can literally stand (as the author did) with one foot in the Atlantic Ocean and the other in the Pacific. The author has observed many mountain passes in the Rocky Mountains and Alps. What connections do mountain passes have with nonlinear partial dif ferential equations? To find out, read on ...

The two-volume set LNCS 6987 and 6988 constitutes the refereed proceedings of the International Conference on Web Information Systems and Mining, WISM 2011, held in Taiyuan, China, in September 2011. The 112 revised full papers presented were carefully reviewed and selected from 472 submissions. The second volume includes 56 papers organized in the following topical sections: management information systems; semantic Web and ontologies; Web content mining; Web information classification; Web information extraction; Web intelligence; Web interfaces and applications; Web services and e-learning; and XML and semi-structured data.

Describes a approach to critical point theory and presents a whole new array of duality and perturbation methods.

A conference was organized to discuss research in variational methods as applied to nonlinear elliptic PDE. This volume resulted from that gathering. Included in this title are both survey and research papers that address important open questions and offer suggestions on analytical and numerical techniques for solving those open problems. It is suitable for graduate students and research mathematicians interested in elliptic partial differential equations.

Many nonlinear problems in physics, engineering, biology and social sciences can be reduced to finding critical points of functionals. While minimax and Morse theories provide answers to many situations and problems on the existence of multiple critical points of a functional, they often cannot provide much-needed additional properties of these critical points. Sign-changing critical point theory has emerged as a new area of rich research on critical points of a differentiable functional with important applications to nonlinear elliptic PDEs. This book is intended for advanced graduate students and researchers involved in sign-changing critical point theory, PDEs, global analysis, and nonlinear functional analysis.

This 2003 book presents min-max methods through a study of the different faces of the celebrated Mountain Pass Theorem (MPT) of Ambrosetti and Rabinowitz. The reader is led from the most accessible results to the forefront of the theory, and at each step in this walk between the hills, the author presents the extensions and variants of the MPT in a complete and unified way. Coverage includes standard topics, but it also covers other topics covered nowhere else in book form: the non-smooth MPT; the geometrically constrained MPT; numerical approaches to the MPT; and even more exotic variants. Each chapter has a section with supplementary comments and bibliographical notes, and there is a rich bibliography and a detailed index to aid the reader. The book is suitable for researchers and graduate students. Nevertheless, the style and the choice of the material make it accessible to all newcomers to the field.

This volume comprises selected papers from the 21st Conference on System Modeling and Optimization in Sophia Antipolis, France. It covers over three decades of studies involving partial differential systems and equations. Topics include: the modeling of continuous mechanics involving fixed boundary, control theory, shape optimization and moving boundaries, and topological shape optimization. This edition discusses all developments that lead to current moving boundary analysis and the stochastic approach.

This book presents some of the latest research in critical point theory, describing methods and presenting the newest applications. Coverage includes extrema, even valued functionals, weak and double linking, sign changing solutions, Morse inequalities, and cohomology groups. Applications described include Hamiltonian systems, Schrödinger equations and systems, jumping nonlinearities, elliptic equations and systems, superlinear problems and beam equations.

A thorough graduate-level introduction to the variational analysis of nonlinear problems described by nonlocal operators.

This book is the second of two volumes that contain the proceedings of the Workshop on Nonlinear Partial Differential Equations, held from May 28-June 1, 2012, at the University of Perugia in honor of Patrizia Pucci's 60th birthday. The workshop brought together leading experts and researchers in nonlinear partial differential equations to promote research and to stimulate interactions among the participants. The workshop program testified to the wide ranging influence of Patrizia Pucci on the field of nonlinear analysis and partial differential equations. In her own work, Patrizia Pucci has been a seminal influence in many important areas: the maximum principle, qualitative analysis of solutions to many classes of nonlinear PDEs (Kirchhoff problems, polyharmonic systems), mountain pass theorem in the critical case, critical exponents, variational identities, as well as various degenerate or singular phenomena in mathematical physics. This same breadth is reflected in the mathematical papers included in this volume. The companion volume (Contemporary Mathematics, Volume 594) is devoted to evolution problems in nonlinear partial differential equations.

The purpose of this book is to present a Morse theoretic study of a very general class of homogeneous operators that includes the p-Laplacian as a special case. The p-Laplacian operator is a quasilinear differential operator that arises in many applications such as non-Newtonian fluid flows and turbulent filtration in porous media. Infinite dimensional Morse theory has been used extensively to study semilinear problems, but only rarely to study the p-Laplacian. The standard tools of Morse theory for computing critical groups, such as the Morse lemma, the shifting theorem, and various linking and local linking theorems based on eigenspaces, do not apply to quasilinear problems where the Euler functional is not defined on a Hilbert space or is not C^2 or where there are no eigenspaces to work with. Moreover, a complete description of the spectrum of a quasilinear operator is generally not available, and the standard sequence of eigenvalues based on the genus is not useful for obtaining nontrivial critical groups or for constructing linking sets and local linkings. However, one of the main points of this book is that the lack of a complete list of eigenvalues is not an insurmountable obstacle to applying critical point theory. Working with a new sequence of eigenvalues that uses the cohomological index, the authors systematically develop alternative tools such as nonlinear linking and local splitting theories in order to effectively apply Morse theory to quasilinear problems. They obtain nontrivial critical groups in nonlinear eigenvalue problems and use the stability and piercing properties of the cohomological index to construct new linking sets and local splittings that are readily applicable here. (SURV/161)

This book reflects a significant part of authors' research activity dur ing the last ten years. The present monograph is constructed on the results obtained by the authors through their direct cooperation or due to the authors separately or in cooperation with other mathematicians. All these results fit in a unitary scheme giving the structure of this work. The book is mainly addressed to researchers and scholars in Pure and Applied Mathematics, Mechanics, Physics and Engineering. We are greatly indebted to Viorica Venera Motreanu for the careful reading of the manuscript and helpful comments on important issues. We are also grateful to our Editors of Kluwer Academic Publishers for their professional assistance. Our deepest thanks go to our numerous scientific collaborators and friends, whose work was so important for us. D. Motreanu and V. Radulescu IX Introduction The present monograph is based on original results obtained by the authors in the last decade. This book provides a comprehensive expo sition of some modern topics in nonlinear analysis with applications to the study of several classes of boundary value problems. Our framework includes multivalued elliptic problems with discontinuities, variational inequalities, hemivariational inequalities and evolution problems. The treatment relies on variational methods, monotonicity principles, topo logical arguments and optimization techniques. Excepting Sections 1 and 3 in Chapter 1 and Sections 1 and 3 in Chapter 2, the material is new in comparison with any other book, representing research topics where the authors contributed. The outline of our work is the following.

This monograph collects cutting-edge results and techniques for solving nonlinear partial differential equations using critical points. Including many of the author's own contributions, a range of proofs are conveniently collected here, Because the material is approached with rigor, this book will serve as an invaluable resource for exploring recent developments in this active area of research, as well as the numerous ways in which critical point theory can be applied. Different methods for finding critical points are presented in the first six chapters. The specific situations in which these methods are applicable is explained in detail. Focus then shifts toward the book's main subject: applications to problems in mathematics and physics. These include topics such as Schrödinger equations, Hamiltonian systems, elliptic systems, nonlinear wave equations, nonlinear optics, semilinear PDEs, boundary value problems, and equations with multiple solutions. Readers will find this collection of applications convenient and thorough, with detailed proofs appearing throughout. Critical Point Theory will be ideal for graduate students and researchers interested in solving differential equations, and for those studying variational methods. An understanding of fundamental mathematical analysis is assumed. In particular, the basic properties of Hilbert and Banach spaces are used.

Publisher Description

It would be hopeless to attempt to give a complete account of the history of the calculus of variations. The interest of Greek philosophers in isoperimetric problems underscores the importance of "optimal form" in ancient cultures, see Hildebrandt-Tromba [1] for a beautiful treatise of this subject. While variatio nal problems thus are part of our classical cultural heritage, the first modern treatment of a variational problem is attributed to Fermat (see Goldstine [1; p.l]). Postulating that light follows a path of least possible time, in 1662 Fer mat was able to derive the laws of refraction, thereby using methods which may already be termed analytic. With the development of the Calculus by Newton and Leibniz, the basis was laid for a more systematic development of the calculus of variations. The brothers Johann and Jakob Bernoulli and Johann's student Leonhard Euler, all from the city of Basel in Switzerland, were to become the "founding fathers" (Hildebrandt-Tromba [1; p.21]) of this new discipline. In 1743 Euler [1] sub mitted "A method for finding curves enjoying certain maximum or minimum properties", published 1744, the first textbook on the calculus of variations.

This volume contains several surveys focused on the ideas of approximate solutions, well-posedness and stability of problems in scalar and vector optimization, game theory and calculus of variations. These concepts are of particular interest in many fields of mathematics. The idea of stability goes back at least to J. Hadamard who introduced it in the setting of differential equations; the concept of well-posedness for minimum problems is more recent (the mid-sixties) and originates with A.N. Tykhonov. It turns out that there are connections between the two properties in the sense that a well-posed problem which, at least in principle, is "easy to solve", has a solution set that does not vary too much under perturbation of the data of the problem, i.e. it is "stable". These themes have been studied in depth for minimum problems and now we have a general picture of the related phenomena in this case. But, of course, the same concepts can be studied in other more complicated situations as, e.g. vector optimization, game theory and variational inequalities. Let us mention that in several of these new areas there is not even a unique idea of what should be called approximate solution, and the latter is at the basis of the definition of well posed problem.

A collection of research articles originating from the Workshop on Nonlinear Analysis and Applications held in Bergamo in July 2001. Classical topics of nonlinear analysis were considered, such as calculus of variations, variational inequalities, critical point theory and their use in various aspects of the study of elliptic differential equations and systems, equations of Hamilton-Jacobi, Schrödinger and Navier-Stokes, and free boundary problems. Moreover, various models were focused upon: travelling waves in supported beams and plates, vortex condensation in electroweak theory, information theory, non-geometrical optics, and Dirac-Fock models for heavy atoms.

This volume is a collection of articles presented at the Workshop for Nonlinear Analysis held in João Pessoa, Brazil, in September 2012. The influence of Bernhard Ruf, to whom this volume is dedicated on the occasion of his 60th birthday, is perceptible throughout the collection by the choice of themes and techniques. The many contributors consider modern topics in the calculus of variations, topological methods and regularity analysis, together with novel applications of partial differential equations. In keeping with the tradition of the workshop, emphasis is given to elliptic operators inserted in different contexts, both theoretical and applied. Topics include semi-linear and fully nonlinear equations and systems with different nonlinearities, at sub- and supercritical exponents, with spectral interactions of Ambrosetti-Prodi type. Also treated are analytic aspects as well as applications such as diffusion problems in mathematical genetics and finance and evolution equations related to electromechanical devices.

This volume covers recent advances in the field of nonlinear functional analysis and its applications to nonlinear partial and ordinary differential equations, with particular emphasis on variational and topological methods. A broad range of topics is covered, including: * concentration phenomena in pdes * variational methods with applications to pdes and physics * periodic solutions of odes * computational aspects in topological methods * mathematical models in biology Though well-differentiated, the topics covered are unified through a common perspective and approach. Unique to the work are several chapters on computational aspects and applications to biology, not usually found with such basic studies on pdes and odes. The volume is an excellent reference text for researchers and graduate students in the above mentioned fields. Contributors: M. Clapp, M. Del Pino, M.J. Esteban, P. Felmer, A. Ioffe, W. Marzantowicz, M. Mrozek, M. Musso, R. Ortega, P. Pilarczyk, E. Séré, E. Schwartzman, P. Sintzoff, R. Turner , M. Willem.

ICM 2002 Satellite Conference on Nonlinear Analysis was held in the period: August 1418, 2002 at Taiyuan, Shanxi Province, China. This conference was organized by Mathematical School of Peking University, Academy of Mathematics and System Sciences of Chinese Academy of Sciences, Mathematical school of Nankai University, and Department of Mathematics of Shanxi University, and was sponsored by Shanxi Province Education Committee, Tian Yuan Mathematics Foundation, and Shanxi University. 166 mathematicians from 21 countries and areas in the world attended the conference. 53 invited speakers and 30 contributors presented their lectures. This conference aims at an overview of the recent development in nonlinear analysis. It covers the following topics: variational methods, topological methods, fixed point theory, bifurcations, nonlinear spectral theory, nonlinear Schrvdinger equations, semilinear elliptic equations, Hamiltonian systems, central configuration in N-body problems and variational problems arising in geometry and physics.

Boundary value problems which have variational expressions in form of inequal ities can be divided into two main classes. The class of boundary value prob lems (BVPs) leading to variational inequalities and the class of BVPs leading to hemivariational inequalities. The first class is related to convex energy functions and has being studied over the last forty years and the second class is related to nonconvex energy functions and has a shorter research "life" beginning with the works of the second author of the present book in the year 1981. Nevertheless a variety of important results have been produced within the framework of the theory of hemivariational inequalities and their numerical treatment, both in Mathematics and in Applied Sciences, especially in Engineering. It is worth noting that inequality problems, i. e. BVPs leading to variational or to hemivariational inequalities, have within a very short time had a remarkable and precipitate development in both Pure and Applied Mathematics, as well as in Mechanics and the Engineering Sciences, largely because of the possibility of applying and further developing new and efficient mathematical methods in this field, taken generally from convex and/or nonconvex Nonsmooth Analy sis. The evolution of these areas of Mathematics has facilitated the solution of many open questions in Applied Sciences generally, and also allowed the formu lation and the definitive mathematical and numerical study of new classes of interesting problems.

This dissertation was to study computational theory and methods for finding multiple saddle critical points in Banach spaces. Two local minimax methods were developed for this purpose. One was for unconstrained cases and the other was for constrained cases. First, two local minmax characterization of saddle critical points in Banach spaces were established. Based on these two local minmax characterizations, two local minimax algorithms were designed. Their flow charts were presented. Then convergence analysis of the algorithms were carried out. Under certain assumptions, a subsequence convergence and a point-to-set convergence were obtained. Furthermore, a relation between the convergence rates of the functional value sequence and corresponding gradient sequence was derived. Techniques to implement the algorithms were discussed. In numerical experiments, those techniques have been successfully implemented to solve for multiple solutions of several quasilinear elliptic boundary value problems and multiple eigenpairs of the well known nonlinear p-Laplacian operator. Numerical solutions were presented by their profiles for visualization. Several interesting phenomena of the solutions of quasilinear elliptic boundary value problems and the eigenpairs of the p-Laplacian operator have been observed and are open for further investigation. As a generalization of the above results, nonsmooth critical points were considered for locally Lipschitz continuous functionals. A local minmax characterization of nonsmooth saddle critical points was also established. To establish its version in Banach spaces, a new notion, pseudo-generalized-gradient has to be introduced. Based on the characterization, a local minimax algorithm for finding multiple nonsmooth saddle critical points was proposed for further study.

The book is intended to be an introduction to critical point theory and its applications to differential equations. Although the related material can be found in other books, the authors of this volume have had the following goals in mind: To present a survey of existing minimax theorems, To give applications to elliptic differential equations in bounded domains, To consider the dual variational method for problems with continuous and discontinuous nonlinearities, To present some elements of critical point theory for locally Lipschitz functionals and give applications to fourth-order differential equations with discontinuous nonlinearities, To study homoclinic solutions of differential equations via the variational methods. The contents of the book consist of seven chapters, each one divided into several sections. Audience: Graduate and post-graduate students as well as specialists in the fields of differential equations, variational methods and optimization.

An examination of progress in mathematical control theory applications. It provides analyses of the influence and relationship of nonlinear partial differential equations to control systems and contains state-of-the-art reviews, including presentations from a conference co-sponsored by the National Science Foundation, the Institute of Mathematics a

This book emphasizes those basic abstract methods and theories that are useful in the study of nonlinear boundary value problems. The content is developed over six chapters, providing a thorough introduction to the techniques used in the variational and topological analysis of nonlinear boundary value problems described by stationary differential operators. The authors give a systematic treatment of the basic mathematical theory and constructive methods for these classes of nonlinear equations as well as their applications to various processes arising in the applied sciences. They show how these diverse topics are connected to other important parts of mathematics, including topology, functional analysis, mathematical physics, and potential theory. Throughout the book a nice balance is maintained between rigorous mathematics and physical applications. The primary readership includes graduate students and researchers in pure and applied nonlinear analysis.

Using minimax methods, some existence theorems are proved for critical points of a real valued function on a Banach space. The critical points are of a saddle point type. Applications are made to semilinear elliptic partial differential equations. A perturbation theorem for the critical points is also proved in this context. Lastly applications are made to a family of nonlinear wave equations.

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The papers collected in this volume are contributions to the 33rd session of the Seminaire de Mathematiques Superieures (SMS) on "Topological Methods in Differential Equations and Inclusions". This session of the SMS took place at the Universite de Montreal in July 1994 and was a NATO Advanced Study Institute (ASI). The aim of the ASI was to bring together a considerable group of young researchers from various parts of the world and to present to them coherent surveys of some of the most recent advances in this area of Nonlinear Analysis. During the meeting 89 mathematicians from 20 countries have had the opportunity to get acquainted with various aspects of the subjects treated in the lectures as well as the chance to exchange ideas and learn about new problems arising in the field. The main topics teated in this ASI were the following: Fixed point theory for single- and multi-valued mappings including topological degree and its generalizations, and topological transversality theory; existence and multiplicity results for ordinary differential equations and inclusions; bifurcation and stability problems; ordinary differential equations in Banach spaces; second order differential equations on manifolds; the topological structure of the solution set of differential inclusions; effects of delay perturbations on dynamics of retarded delay differential equations; dynamics of reaction diffusion equations; non smooth critical point theory and applications to boundary value problems for quasilinear elliptic equations.