Mathematical Logic

David Hilbert was particularly interested in the foundations of mathematics. Among many other things, he is famous for his attempt to axiomatize mathematics. This now classic text is his treatment of symbolic logic. This translation is based on the second German edition and has been modified according to the criticisms of Church and Quine. In particular, the authors' original formulation of Godel's completeness proof for the predicate calculus has been updated. In the first half of the twentieth century, an important debate on the foundations of mathematics took place. Principles of Mathematical Logic represents one of Hilbert's important contributions to that debate. Although symbolic logic has grown considerably in the subsequent decades, this book remains a classic.

This comprehensive overview ofmathematical logic is designedprimarily for advanced undergraduatesand graduate studentsof mathematics. The treatmentalso contains much of interest toadvanced students in computerscience and philosophy. Topics include propositional logic;first-order languages and logic; incompleteness, undecidability,and indefinability; recursive functions; computability;and Hilbert’s Tenth Problem.Reprint of the PWS Publishing Company, Boston, 1995edition.

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This volume offers insights into the development of mathematical logic over the last century. Arising from a special session of the history of logic at an American Mathematical Society meeting, the chapters explore technical innovations, the philosophical consequences of work during the period, and the historical and social context in which the logicians worked. The discussions herein will appeal to mathematical logicians and historians of mathematics, as well as philosophers and historians of science.

This book provides a distinctive, well-motivated introduction to mathematical logic. It starts with the definition of first order languages, proceeds through propositional logic, completeness theorems, and finally the two Incompleteness Theorems of Godel.

Rigorous introduction is simple enough in presentation and context for wide range of students. Symbolizing sentences; logical inference; truth and validity; truth tables; terms, predicates, universal quantifiers; universal specification and laws of identity; more.

Contents include an elementary but thorough overview of mathematical logic of 1st order; formal number theory; surveys of the work by Church, Turing, and others, including Gödel's completeness theorem, Gentzen's theorem, more.

This book, presented in two parts, offers a slow introduction to mathematical logic, and several basic concepts of model theory, such as first-order definability, types, symmetries, and elementary extensions. Its first part, Logic Sets, and Numbers, shows how mathematical logic is used to develop the number structures of classical mathematics. The exposition does not assume any prerequisites; it is rigorous, but as informal as possible. All necessary concepts are introduced exactly as they would be in a course in mathematical logic; but are accompanied by more extensive introductory remarks and examples to motivate formal developments. The second part, Relations, Structures, Geometry, introduces several basic concepts of model theory, such as first-order definability, types, symmetries, and elementary extensions, and shows how they are used to study and classify mathematical structures. Although more advanced, this second part is accessible to the reader who is either already familiar with basic mathematical logic, or has carefully read the first part of the book. Classical developments in model theory, including the Compactness Theorem and its uses, are discussed. Other topics include tameness, minimality, and order minimality of structures. The book can be used as an introduction to model theory, but unlike standard texts, it does not require familiarity with abstract algebra. This book will also be of interest to mathematicians who know the technical aspects of the subject, but are not familiar with its history and philosophical background.

The Fourth Edition of this long-established text retains all the key features of the previous editions, covering the basic topics of a solid first course in mathematical logic. This edition includes an extensive appendix on second-order logic, a section on set theory with urlements, and a section on the logic that results when we allow models with empty domains. The text contains numerous exercises and an appendix furnishes answers to many of them. Introduction to Mathematical Logic includes: propositional logic first-order logic first-order number theory and the incompleteness and undecidability theorems of Gödel, Rosser, Church, and Tarski axiomatic set theory theory of computability The study of mathematical logic, axiomatic set theory, and computability theory provides an understanding of the fundamental assumptions and proof techniques that form basis of mathematics. Logic and computability theory have also become indispensable tools in theoretical computer science, including artificial intelligence. Introduction to Mathematical Logic covers these topics in a clear, reader-friendly style that will be valued by anyone working in computer science as well as lecturers and researchers in mathematics, philosophy, and related fields.

In the recent decades mathematical logic has become more and more important in computer science and, in general, in system engineering. In fact, by definition, it is the wey of expressing our reasoning in terms of mathematical formalism, thus supplying it with the typical rigor and precision of mathematics. Not by chance, automatic information processing is now pervasive and we find it practically in any human activity and artefact, from embedded, safety-critical systems, to e-commerce, to social networks, etc. Such a pervasiveness and the consequent heterogeneity of the involved systems mandate much more generality in the formalism supporting the engineering activity than traditional specialized models such as, e.g., those for electric circuits and mechanical engines: mathematical logic, paired with computer applications, provides such generality

This book was written to serve as an introduction to logic, with in each chapter – if applicable – special emphasis on the interplay between logic and philosophy, mathematics, language and (theoretical) computer science. The reader will not only be provided with an introduction to classical logic, but to philosophical (modal, epistemic, deontic, temporal) and intuitionistic logic as well. The first chapter is an easy to read non-technical Introduction to the topics in the book. The next chapters are consecutively about Propositional Logic, Sets (finite and infinite), Predicate Logic, Arithmetic and Gödel's Incompleteness Theorems, Modal Logic, Philosophy of Language, Intuitionism and Intuitionistic Logic, Applications (Prolog; Relational Databases and SQL; Social Choice Theory, in particular Majority Judgment) and finally, Fallacies and Unfair Discussion Methods. Throughout the text, the author provides some impressions of the historical development of logic: Stoic and Aristotelian logic, logic in the Middle Ages and Frege's Begriffsschrift, together with the works of George Boole (1815-1864) and August De Morgan (1806-1871), the origin of modern logic. Since "if ..., then ..." can be considered to be the heart of logic, throughout this book much attention is paid to conditionals: material, strict and relevant implication, entailment, counterfactuals and conversational implicature are treated and many references for further reading are given. Each chapter is concluded with answers to the exercises. Philosophical and Mathematical Logic is a very recent book (2018), but with every aspect of a classic. What a wonderful book! Work written with all the necessary rigor, with immense depth, but without giving up clarity and good taste. Philosophy and mathematics go hand in hand with the most diverse themes of logic. An introductory text, but not only that. It goes much further. It's worth diving into the pages of this book, dear reader! Paulo Sérgio Argolo

A serious introductory treatment geared toward non-logicians, this survey traces the development of mathematical logic from ancient to modern times and discusses the work of Planck, Einstein, Bohr, Pauli, Heisenberg, Dirac, and others. 1972 edition.

Written by a creative master of mathematical logic, this introductory text combines stories of great philosophers, quotations, and riddles with the fundamentals of mathematical logic. Author Raymond Smullyan offers clear, incremental presentations of difficult logic concepts. He highlights each subject with inventive explanations and unique problems. Smullyan's accessible narrative provides memorable examples of concepts related to proofs, propositional logic and first-order logic, incompleteness theorems, and incompleteness proofs. Additional topics include undecidability, combinatoric logic, and recursion theory. Suitable for undergraduate and graduate courses, this book will also amuse and enlighten mathematically minded readers. Dover (2014) original publication. See every Dover book in print at www.doverpublications.com

This is a mathematics textbook with theorems and proofs. The choice of topics has been guided by the needs of computer science students. The method of semantic tableaux provides an elegant way to teach logic that is both theoretically sound and yet sufficiently elementary for undergraduates. In order to provide a balanced treatment of logic, tableaux are related to deductive proof systems. The book presents various logical systems and contains exercises. Still further, Prolog source code is available on an accompanying Web site. The author is an Associate Professor at the Department of Science Teaching, Weizmann Institute of Science.

At the intersection of mathematics, computer science, and philosophy,

mathematical logic examines the power and limitations of formal mathematical thinking. In this expansion of Leary's user-friendly 1st edition, readers with no previous study in the field are introduced to the basics of model theory, proof theory, and computability theory. The text is designed to be used either in an upper division undergraduate classroom, or for self study. Updating the 1st Edition's treatment of languages, structures, and deductions, leading to rigorous proofs of Godel's First and Second Incompleteness Theorems, the expanded 2nd Edition includes a new introduction to incompleteness through computability as well as solutions to selected exercises.
Mathematical Logic is a collection of the works of one of the leading figures in 20th-century science. This collection of A.M. Turing's works is intended to include all his mature scientific writing, including a substantial quantity of unpublished material. His work in pure mathematics and mathematical logic extended considerably further; the work of his last years, on morphogenesis in plants, is also of the greatest originality and of permanent importance. This book is divided into three parts. The first part focuses on computability and ordinal logics and covers Turing's work between 1937 and 1938. The second part covers type theory; it provides a general introduction to Turing's work on type theory and covers his published and unpublished works between 1941 and 1948. Finally, the third part focuses on enigmas, mysteries, and loose ends. This concluding section of the book discusses Turing's Treatise on the Enigma, with excerpts from the Enigma Paper. It also delves into Turing's papers on programming and on minimum cost sequential analysis, featuring an excerpt from the unpublished manuscript. This book will be of interest to mathematicians, logicians, and computer scientists.
A mathematical introduction to the theory and applications of logic and set theory with an emphasis on writing proofs Highlighting the applications and notations of basic mathematical concepts within the framework of logic and set theory, A First Course in Mathematical Logic and Set Theory introduces how logic is used to prepare and structure proofs and solve more complex problems. The book begins with propositional logic, including two-column proofs and truth table applications, followed by first-order logic, which provides the structure for writing mathematical proofs. Set theory is then introduced and serves as the basis for defining relations, functions, numbers, mathematical induction, ordinals, and cardinals. The book concludes with a primer on basic model theory with applications to abstract algebra. A First Course in Mathematical Logic and Set Theory also includes: Section exercises designed to show the interactions between topics and reinforce the presented ideas and concepts Numerous examples that illustrate theorems and employ basic concepts such as Euclid’s lemma, the Fibonacci sequence, and unique factorization Coverage of important theorems including the well-ordering theorem, completeness theorem, compactness theorem, as well as the theorems of Löwenheim–Skolem, Burali-Forti, Hartogs, Cantor–Schröder–Bernstein, and König An excellent textbook for

students studying the foundations of mathematics and mathematical proofs, A First Course in Mathematical Logic and Set Theory is also appropriate for readers preparing for careers in mathematics education or computer science. In addition, the book is ideal for introductory courses on mathematical logic and/or set theory and appropriate for upper-undergraduate transition courses with rigorous mathematical reasoning involving algebra, number theory, or analysis.
Before his death in March, 1976, A. H. Lightstone delivered the manu script for this book to Plenum Press. Because he died before the editorial work on the manuscript was completed, I agreed (in the fall of 1976) to serve as a surrogate author and to see the project through to completion. I have changed the manuscript as little as possible, altering certain passages to correct oversights. But the alterations are minor; this is Lightstone's book. H. B. Enderton vii Preface This is a treatment of the predicate calculus in a form that serves as a foundation for nonstandard analysis. Classically, the predicates and variables of the predicate calculus are kept distinct, inasmuch as no variable is also a predicate; moreover, each predicate is assigned an order, a unique natural number that indicates the length of each tuple to which the predicate can be prefixed. These restrictions are dropped here, in order to develop a flexible, expressive language capable of exploiting the potential of nonstandard analysis. To assist the reader in grasping the basic ideas of logic, we begin in Part I by presenting the propositional calculus and statement systems. This provides a relatively simple setting in which to grapple with the some times foreign ideas of mathematical logic. These ideas are repeated in Part II, where the predicate calculus and semantical systems are studied.

The handbook is divided into four parts: model theory, set theory, recursion theory and proof theory. Each of the four parts begins with a short guide to the chapters that follow. Each chapter is written for non-specialists in the field in question. Mathematicians will find that this book provides them with a unique opportunity to apprise themselves of developments in areas other than their own.
This lucid, non-intimidating presentation by a Russian scholar explores propositional logic, propositional calculus, and predicate logic. Topics include computer science and systems analysis, linguistics, and problems in the foundations of mathematics. Accessible to high school students, it also constitutes a valuable review of fundamentals for professionals. 1970 edition.
A comprehensive one-year graduate (or advanced undergraduate) course in mathematical logic and foundations of mathematics. No previous knowledge of logic is required; the book is suitable for self-study. Many exercises (with hints) are included.

This introduction to first-order logic clearly works out the role of first-order logic in the foundations of mathematics, particularly the two basic questions of the range of the axiomatic method and of theorem-proving by machines. It covers several advanced topics not commonly treated in introductory texts, such as Fraïssé's characterization of elementary equivalence, Lindström's theorem on the

maximality of first-order logic, and the fundamentals of logic programming.
This introduction to mathematical logic explores philosophical issues and Gödel's Theorem. Its widespread influence extends to the author of Gödel, Escher, Bach, whose Pulitzer Prize–winning book was inspired by this work.
Mathematical logic is a branch of mathematics that takes axiom systems and mathematical proofs as its objects of study. This book shows how it can also provide a foundation for the development of information science and technology. The first five chapters systematically present the core topics of classical mathematical logic, including the syntax and models of first-order languages, formal inference systems, computability and representability, and Gödel’s theorems. The last five chapters present extensions and developments of classical mathematical logic, particularly the concepts of version sequences of formal theories and their limits, the system of revision calculus, proschemes (formal descriptions of proof methods and strategies) and their properties, and the theory of inductive inference. All of these themes contribute to a formal theory of axiomatization and its application to the process of developing information technology and scientific theories. The book also describes the paradigm of three kinds of language environments for theories and it presents the basic properties required of a meta-language environment. Finally, the book brings these themes together by describing a workflow for scientific research in the information era in which formal methods, interactive software and human invention are all used to their advantage. This book represents a valuable reference for graduate and undergraduate students and researchers in mathematics, information science and technology, and other relevant areas of natural sciences. Its first five chapters serve as an undergraduate text in mathematical logic and the last five chapters are addressed to graduate students in relevant disciplines.
Heyting'88 Summer School and Conference on Mathematical Logic, held September 13 - 23, 1988 in Chaika, Bulgaria, was honourably dedicated to Arend Heyting's 90th anniversary. It was organized by Sofia University "Kliment Ohridski" on the occasion of its centenary and by the Bulgarian Academy of Sciences, with sponsorship of the Association for Symbolic Logic. The Meeting gathered some 115 participants from 19 countries. The present volume consists of invited and selected papers. Included are all the invited lectures submitted for publication and the 14 selected contributions, chosen out of 56 submissions by the Selection Committee. The selection was made on the basis of reports of PC members, an average of 4 per sLlbmission. All the papers are concentrated on the topics of the Meeting: Recursion Theory, Modal and Non-classical Logics, Intuitionism and Constructivism, Related Applications to Computer and Other Sciences, Life and Work of Arend Heyting. I am pleased to thank all persons and institutions that contributed to the success of the Meeting: sponsors, Programme Committee members and additional referees, the members of the Organizing Committee, our secretaries K. Lozanova and L. Nikolova, as well as K. Angelov, V. Bozhichkova, A. Ditchev, D. Dobrev, N. Dimitrov, R. Draganova, G. Gargov, N.

Georgieva, M. Janchev, P. Marinov, S. Nikolova, S. Radev, I. Soskov, A. Soskova and v. Sotirov, who helped in the organization, Plenum Press and at last but not least all participants in the Meeting and contributors to this volume.

Mathematical LogicCourier Corporation

Written by a pioneer of mathematical logic, this comprehensive graduate-level text explores the constructive theory of first-order predicate calculus. It covers formal methods — including algorithms and epitheory — and offers a brief treatment of Markov's approach to algorithms. It also explains elementary facts about lattices and similar algebraic systems. 1963 edition.

Mathematical logic developed into a broad discipline with many applications in mathematics, informatics, linguistics and philosophy. This text introduces the fundamentals of this field, and this new edition has been thoroughly expanded and revised.

"Attractive and well-written introduction." — Journal of Symbolic Logic The logic that mathematicians use to prove their theorems is itself a part of mathematics, in the same way that algebra, analysis, and geometry are parts of mathematics. This attractive and well-written introduction to mathematical logic is aimed primarily at undergraduates with some background in college-level mathematics; however, little or no acquaintance with abstract mathematics is needed. Divided into three chapters, the book begins with a brief encounter of naïve set theory and logic for the beginner, and proceeds to set forth in elementary and intuitive form the themes developed formally and in detail later. In Chapter Two, the predicate calculus is developed as a formal axiomatic theory. The statement calculus, presented as a part of the predicate calculus, is treated in detail from the axiom schemes through the deduction theorem to the completeness theorem. Then the full predicate calculus is taken up again, and a smooth-running technique for proving theorem schemes is developed and exploited. Chapter Three is devoted to first-order theories, i.e., mathematical theories for which the predicate calculus serves as a base. Axioms and short developments are given for number theory and a few algebraic theories. Then the metamathematical notions of consistency, completeness, independence, categoricity, and decidability are discussed, The predicate calculus is proved to be complete. The book concludes with an outline of Godel's incompleteness theorem. Ideal for a one-semester course, this concise text offers more detail and mathematically relevant examples than those available in elementary books on logic. Carefully chosen exercises, with selected answers, help students test their grasp of the material. For any student of mathematics, logic, or the interrelationship of the two, this book represents a thought-provoking introduction to the logical underpinnings of mathematical theory. "An excellent text." — Mathematical Reviews

8.3 The consistency proof -- 8.4 Applications of the consistency proof -- 8.5 Second-order arithmetic -- Problems -- Chapter 9: Set Theory -- 9.1 Axioms for sets -- 9.2 Development of set theory -- 9.3 Ordinals -- 9.4 Cardinals -- 9.5 Interpretations of set theory -- 9.6 Constructible sets -- 9.7 The axiom of constructibility -- 9.8 Forcing -- 9.9 The independence proofs -- 9.10 Large cardinals -- Problems -- Appendix The Word Problem -- Index

This self-contained text will appeal to readers from diverse fields and varying backgrounds. Topics include 1st-order recursive arithmetic, 1st- and 2nd-order logic, and the arithmetization of syntax. Numerous exercises; some solutions. 1969 edition.

Assuming no previous study in logic, this informal yet rigorous text covers the material of a standard undergraduate first course in mathematical logic, using natural deduction and leading up to the completeness theorem for first-order logic. At each stage of the text, the reader is given an intuition based on standard mathematical practice, which is subsequently developed

with clean formal mathematics. Alongside the practical examples, readers learn what can and can't becalculated; for example the correctness of a derivation proving a given sequent can be tested mechanically, but there is no general mechanical test for the existence of a derivation proving the given sequent. The undecidability results are proved rigorously in an optional final chapter, assumingMatiyasevich's theorem characterising the computably enumerable relations. Rigorous proofs of the adequacy and completeness proofs of the relevant logics are provided, with careful attention to the languages involved. Optional sections discuss the classification of mathematical structures by first-order theories; the required theory of cardinality is developed from scratch. Throughout the book there are notes on historical aspects of the material, and connections with linguistics andcomputer science, and the discussion of syntax and semantics is influenced by modern linguistic approaches. Two basic themes in recent cognitive science studies of actual human reasoning are also introduced. Including extensive exercises and selected solutions, this text is ideal for students in Logic,Mathematics, Philosophy, and Computer Science.
The Summer School and Conference on Mathematical Logic and its Applications, September 24 - October 4, 1986, Druzhba, Bulgaria, was honourably dedicated to the 80-th anniversary of Kurt Godel (1906 - 1978), one of the greatest scientists of this (and not only of this) century. The main topics of the Meeting were: Logic and the Foundation of Mathematics; Logic and Computer Science; Logic, Philosophy, and the Study of Language; Kurt Godel's life and deed. The scientific program comprised 5 kinds of activities, namely: a) a Godel Session with 3 invited lecturers b) a Summer School with 17 invited lecturers c) a Conference with 13 contributed talks d) Seminar talks (one invited and 12 with no preliminary selection) e) three discussions The present volume reflects an essential part of this program, namely 14 of the invited lectures and all of the contributed talks. Not presented in the volltme remai ned si x of the i nvi ted lecturers who di d not submi t texts: Yu. Ershov - The Language of !:-expressions and its Semantics; S. Goncharov - Mathematical Foundations of Semantic Programming; Y. Moschovakis - Foundations of the Theory of Algorithms; N. Nagornyj - Is Realizability of Propositional Formulae a GBdelean Property; N. Shanin - Some Approaches to Finitization of Mathematical Analysis; V. Uspensky - Algorithms and Randomness - joint with A. N.
In case you are considering to adopt this book for courses with over 50 students, please contact ties.nijssen@springer.com for more information. This introduction to mathematical logic starts with propositional calculus and first-order logic. Topics covered include syntax, semantics, soundness, completeness, independence, normal forms, vertical paths through negation normal formulas, compactness, Smullyan's Unifying Principle, natural deduction, cut-elimination, semantic tableaux, Skolemization, Herbrand's Theorem, unification, duality, interpolation, and definability. The last three chapters of the book provide an introduction to type theory (higher-order logic). It is shown how various mathematical concepts can be formalized in this very expressive formal language. This expressive notation facilitates proofs of the classical incompleteness and undecidability theorems which are very elegant and easy to understand. The discussion of semantics makes clear the important distinction between standard and nonstandard models which is so important in understanding puzzling phenomena such as the incompleteness theorems and Skolem's Paradox about countable models of set theory. Some of the numerous exercises require giving formal proofs. A computer program called ETPS which is available from the web facilitates doing and checking such exercises. Audience: This volume will be of interest to mathematicians, computer scientists, and philosophers in universities, as well as to computer scientists in industry who wish to use higher-order logic for hardware and software specification and verification.