Mathematical Economics Lecture Notes

This monograph is devoted to a rigorous presentation of results about the representation of preference orderings by real-valued order isomorphisms ("utility functions"). The authors have gathered together, from sources scattered throughout a wide range of the literature of mathematics and economics, the most significant theorems and methods in their field. The book will be a valuable resource for theoretical economists, mathematicians, and all interested in partial orders and their representation. It could also be used as the text for an advanced graduate course in mathematical economics.
These essays in honor of Professor Gerhard Tintner are substantive contributions to three areas of econometrics, (1) economic models and applications,. (2) estimation, and (3) stochastic programming, in each of which he has labored with outstanding success. His own work has extended into multivariate analysis, the pure theory of decision-making under un certainty, and other fields which are not touched upon here for reasons of space and focus. Thus, this collection is appropriate to his interests but covers much less than their full range. Professor Tintner's contributions to econometrics through teaching, writing, editing, lecturing and consulting have been varied and inter national. We have tried to highlight them in "The Econometric Work of Gerhard Tintner" and to place them in historical perspective in "The Invisible Revolution in Economics: Emergence of a Mathematical Science. " Professor Tintner's career to date has spanned the organizational life of the Econometric Society and his contributions have been nearly coextensive with its scope. His principal books and articles up to 1968 are listed in the "Selected Bibliography. " Professor Tintner's current research involves the intricate problems of specification and application of stochastic processes to economic systems, particularly to growth, diffusion of technology, and optimal control. As always, he is moving with the econometric frontier and a portion of the frontier is moving with him. IV Two of the editors wrote dissertations under Professor Tintner's sup- vision; the third knew him as a colleague and friend.
The following lecture notes were written shortly after I gave a course on capital theory in the winter-semester 1970/71 at the Univer sity of Heidelberg. While the general line of the argument is similar to the one in the course, I have modified and added a large number' of specific points in the process of writing the English version. I should like to emphasize the narrow limitations of the material covered in these notes. I have completely concentrated on steady states of stationary and exponentially growing economies, even up to the point where there is the danger of misleading the reader1 I have done this for several reasons. Other activities have not left me with a sufficient amount of time to be able to find the unifying principle of analysis and mode of presentation for the dynamic aspects of capi tal theory which would have made it worthwhile to add a sizeable book to the large body of literature in this field. On the other hand over the last couple of

years I have become increasingly aware that some of the results in steady state capital theory (which could be derived without too much mathematical effort) are of relevance in present day dis cussions about the political role of economic theory and the relative merits of orthodox and radical economics. Also these results seemed not to be known by' mO$ of the participants in these discussions.

The role of asymmetric information in allocation of resources, together with the associated information-revelation process, has long been a central focus of economic research. While the bulk of the literature addresses these is sues within the framework of principal-agent relationship, which essentially reduces the problem to the sole principal's (the sole Stackelberg leader's) optimization problem subject to the agents' (the Stackelberg followers') re sponses, there are recent attempts to extend analysis to other economic setups characterized by different relationships among decision-makers. A notable strand of such attempts is the core analysis of incomplete in formation. Here, there is no Stackelberg-type relationship, and more impor tantly the players can talk to each other for coordinated choice of strategies. See, e.g., Wilson (1978) for a pioneering work; Yannelis (1991) for formula tion of feasibility of a strategy as its measurability; Ichiishi and Idzik (1996) for introduction of Bayesian incentive-compatibility to this strand; Ichiishi, Idzik and Zhao (1994) for information revelation (that is, endogenous deter mination of updated information structures); Ichiishi and Radner (1997) and Ichiishi and Sertel (1998) for studies of a specific model of Chandler's firm in multidivisional form for sharper results; and Vohra (1999) for a recent work. It is a common postulate in these works that every player takes part in design of a mechanism and also in execution of the signed contract.

Steady State Capital TheorySpringer

This book is devoted to the study of dynamical models of decentralized economic systems. The models considered are based on the Leontief simple dynamic model with various mechanisms for decentralized planning and management. Branches of the economic system are treated as fully independent economic agents that plan their work according to their own purposes. It is shown that the lack of coordination between economic agents leads to a limit cycle for some economic indicators. Conversely, the exchange of information between the economic agents enables a move toward balanced growth. These results are generalized for the model with dynamics of the productive assets and for the model with the final consumption. The analysis also considers a problem of endogenous technological progress in a decentralized economy. The appendix includes a short review of non-negative matrices. The book offers a valuable resource for mathematical economists and graduate students specializing in mathematical economics.

In a wide number of economic problems the equilibrium values of the variables can be regarded as solutions of a parametrized constrained maximization problem. This occurs in static as well as dynamic models; in the latter case the choice variables are often paths in certain function spaces and thus can be regarded as points in infinite dimensional spaces. It is sometimes possible to determine qualitative properties of the solutions with respect to changes in the parameters of the model. The study of such properties is often called comparative statics; [15], [2], and [10]. Certain comparative static properties of the maxima have proven to be of particular importance for economic theory, since the works of Slutsky, Hicks, and Samuelson [15]: they have been for- lated in terms of synunetry and negative semidefiniteness of a matrix, called the Slutsky-Hicks-Samuelson matrix. A discussion of this matrix and its applications is given in Section 1. The study of these properties in economic theory, however, has so far been restricted to static models where the choice variable and the parameters are elements in Euclidean spaces, and where there is only one constraint.

In winter 71/72 I held a seminar on general equilibrium theory for a jOint group of students in mathematics and in econo mics at the university of Bonn , w.Germany1~ The economists , how ever , had a mathematical background well above the average • Most of the material treated in that seminar is described in these notes. The connection between smooth preferences and smooth demand func tions [see Debreu (1972)] and regular economies based on agents with smooth preferences are not presented here • Some pedagogical difficulties arose from the fact that elementary knowledge of algebraic topology is not assumed although it is helpful and indeed necessary to make some arguments precise • It is only a minor restriction , at present , that functional ana lysis is not used • But with the development of the theory more economic questions will be considered in their natural infinite dimensional setting • Economic knowledge is not required , but especially a reader without economic background will gain much by reading Debreu's classic "Theory of Value" (1959) • Although the formulation of our economic problem uses a map between Euclidean spaces only , we shall also consider ma- folds • Manifolds appear in our situation because inverse images under differentiable mappings between Euclidean spaces are very often differentiable manifolds • (Under differentiability assump tions , for instance , the graph of the equilibrium set correspon

Let eRN be the usual vector-space of real N-uples with the usual inner product denoted by (. ,.). In this paper P is a nonempty compact polyhedral set of mN, f is a real-valued function defined on (RN continuously differentiable and fP is the line- ly constrained minimization problem stated as : min (f(x) I x € P) • For computing stationary points of problemtj) we propose a method which attempts to operate within the linear-simplex method structure. This method then appears as a same type of method as the convex-simplex method of Zangwill [6]. It is however, different and has the advantage of being less technical with regards to the Zangwill method. It has also a simple geometrical interpretation which makes it more under standable and more open to other improvements. Also in the case where f is convex an implementable line-search is proposed which is not the case in the Zangwill method. Moreover, if f(x) = (c,x) this method will coincide with the simplex method (this is also true in the case of the convex simplex method) i if f(x) = I Ixl 12 it will be almost the same as the algorithm given by Bazaraa, Goode, Rardin [2].

This paper is intended for the beginner. It is not a state of-the-art paper for research workers in the field of control theory. Its purpose is to introduce the reader to some of the problems and results in control theory, to illustrate the application of these re sults, and to provide a guide for his further reading on this subject. I have tried to motivate the results with examples, especial ly with one canonical, simple example described in §3. Many results, such as the maximum principle, have long and difficult proofs. I have omitted these proofs. In general I have included only the proofs which are either (1) not too difficult or (2) fairly enlightening as to the nature of the result. I have, however, usually attempted to draw the strongest conclusion from a given proof. For example, many existing proofs in control theory for compact targets and uniqueness of solutions also hold for closed targets and non-uniqueness. Finally, at the end of each section I have given references to generalizations and origins of the results discussed in that section. I make no claim of completeness in the references, however, as I have often been content merely to refer the reader either to an exposition or to a paper which has an extensive bibliography. IV These 1ecture notes are revisions of notes I used for aseries of nine 1ectures on contro1 theory at the International Summer Schoo1 on Mathematica1 Systems and Economics held in Varenna, Ita1y, June 1967.

Presentation Many economic problems, as equilibrium models, input-output analysis, rational behaviour, etc. , are usually modelled in terms of operators in Euclidean spaces. This monograph deals with the analysis of a number of formal problems involving this kind of operators (with particular reference to complementarity problems and variational inequalities), and their applications to distributive problems and equilibrium models. Thus the purpose of this work is to provide a set of new results on the solvability of those problems, and a number of economic applications that will illustrate the interest of these results in economics. It is worth stressing from the very begining that our analysis concentrates on the existence (and in some cases optimality) of solutions. That is what is meant here by solvability (in particular, nothing will be said with respect to the uniqueness, stability, sensitivity analysis or computation of solutions). The results on the solvability of operator problems presented here, were actually arrived at as a way of solving specific economic models. Yet we are going to relate this case by somehow reversing the way it happened, that is, starting with the formal results and then presenting a number of economic models which appear as applications of VIII these formal results. The rationale for this approach is twofold. First, it provides a neat track via which to go through the whole work. Then, because I would like to emphasize the interest of complementarity and variational inequalities problems in economic modelling.

Multiple-output production functions; Functional forms for multiple-output technologies; Stochastic specification; Empirical results.

The International Summer School on Mathematical Systems Theory and Economics was held at the Villa Monastero in Varenna, Italy, from June 1 through June 12, 1967. The objective of this Summer School was to review the state of the art and the prospects for the application of the mathematical theory of systems to the study and the solution of economic problems. Particular emphasis was given to the use of the mathematical theory of control for the solution of problems in economics. It was felt that the publication of a volume collecting most of the lectures given at the school would show the current status of the application of these methods. The papers are organized into four sections arranged into two volumes: basic theories and optimal control of economic systems which appear in the first volume, and special mathematical problems and special applications which are contained in the second volume. Within each section the papers follow in alphabetical order by author. The seven papers on basic theories are a rather

complete representative sample of the fundaments of general systems theory, of the theory of dynamical systems and the theory of control. The five papers on the application of optimal control to economic systems present a broad spectrum of applications.
On February 20, 1978, the Department of Econometrics of the University of Tilburg organized a symposium on Convex Analysis and Mathematical th Economics to commemorate the 50 anniversary of the University. The general theme of the anniversary celebration was "innovation" and since an important part of the departments' theoretical work is con centrated on mathematical economics, the above mentioned theme was chosen. The scientific part of the Symposium consisted of four lectures, three of them are included in an adapted form in this volume, the fourth lec ture was a mathematical one with the title "On the development of the application of convexity". The three papers included concern recent developments in the relations between convex analysis and mathematical economics. Dr. P.H.M. Ruys and Dr. H.N. Weddepohl (University of Tilburg) study in their paper "Economic theory and duality", the relations between optimality and equilibrium concepts in economic theory and various duality concepts in convex analysis. The models are introduced with an individual facing a decision in an optimization problem. Next, an n person decision problem is analyzed, and the following concepts are defined: optimum, relative optimum, Nash-equilibrium, and Pareto-optimum.
This work was written during my visits at CORE (Belgium), at the Faculty of Economics and Politics in Cambridge (England), and at the Department of Mathematics at the ETH in ZUrich. I wish to thank J.H. Dr~ze (CORE) for most help ful suggestions, and I am indebted to H. BUhlmann (ETH) for his advice and for encouragement. The comments by M. Granzio1, M. Janssen and by anonymous referees were very useful. However, I assume the responsibility for remaining errors. I am grateful to R. Boller, I. Lather and M. Urfer for their careful typing of the manuscript. Support by the SWiss National Science Foundation is acknowledged. An earlier version of this work was presented at the European Meeting of the Econometric Society 1981 in Amsterdam. In 1981 it was accepted as a "Habi1itations schrift" in Mathematical Economics by the Department of Mathematics at the ETH in ZUrich. CONTENTS Introduction 1 Part I: The General Model 8 1.1. Formulation of the General Model 8 1.1.1. General Properties 8 1.1.2. The Consumer 10 1.1.3. The Producer 14 1.1.4. The Public Sector 16 Equilibrium 1.2.
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This book is based on lectures given by the author at the IBM European Systems Research Institute (ESRI) in Geneva. Information Theory on the syntactic level, as introduced by Claude Shannon in 1949, has many limitations when applied to information processing by computers. But in spite of some obvious shortcomings, the underlyin~ principles are of fundamental importance for systems engineers in understanding the nature of the problems of handling information, its acquisition, storage, processing, and interpretation. The lectures, as presented in this book, attempt to give an exposition of the lovical foundation and basic principles, and to provide at the same time a basis for further study in more specific areas of this expan1in~ theory, such as coding, detection, pattern recognition, and filtering. Most of the problems in Appendix C are intended as extensions of the text, while calling for actjve participation by the stu1ent. Some other problems are direct applications of the theory to specific situations.

Some problems require extensive numerical calculations. It is assumed in those cases that the student has access to a computer and that he is capable of writing the necessary programs. The stu1ent is assumed to have a good command of the calculus, and of the theory of probability as well as statistics. Therefore no basic mathematical concepts are discussed in this IV book. The Fourier transform and some related mathematical concepts are introduced in Appendix A.

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