From Calculus To Cohomology De Rham Cohomology And Characteristic Classes

Perhaps the most famous example of how ideas from modern physics have revolutionized mathematics is the way string theory has led to an overhaul of enumerative geometry, an area of mathematics that started in the eighteen hundreds. Century-old problems of enumerating geometric configurations have now been solved using new and deep mathematical techniques inspired by physics!The book begins with an insightful introduction to enumerative geometry. From there, the goal becomes explaining the more advanced elements of enumerative algebraic geometry. Along the way, there are some crash courses on intermediate topics which are essential tools for the student of modern mathematics, such as cohomology and other topics in geometry. The physics content assumes nothing beyond a first undergraduate course. The focus is on explaining the action principle in physics, the idea of string theory, and how these directly lead to questions in geometry. Once these topics are in place, the connection between physics and enumerative geometry is made with the introduction of topological quantum field theory and quantum cohomology.

This book represents a novel approach to differential topology. Its main focus is to give a comprehensive introduction to the classification of manifolds, with special attention paid to the case of surfaces, for which the book provides a complete classification from many points of view: topological, smooth, constant curvature, complex, and conformal. Each chapter briefly revisits basic results usually known to graduate students from an alternative perspective, focusing on surfaces. We provide full proofs of some remarkable results that sometimes are missed in basic courses (e.g., the construction of triangulations on surfaces, the classification of surfaces, the Gauss-Bonnet theorem, the degree-genus formula for complex plane curves, the existence of constant curvature metrics on conformal surfaces), and we give hints to questions about higher dimensional manifolds. Many examples and remarks are scattered through the book. Each chapter ends with an exhaustive collection of problems and a list of topics for further study. The book is primarily addressed to graduate students who did take standard introductory courses on algebraic topology, differential and Riemannian geometry, or algebraic geometry, but have not seen their deep interconnections, which permeate a modern approach to geometry and topology of manifolds.

This volume contains the Proceedings of the 3rd IFToMM Symposium on Mechanism Design for Robotics, held in Aalborg, Denmark, 2-4 June, 2015. The book contains papers on recent advances in the design of mechanisms and their robotic applications. It treats the following topics: mechanism design, mechanics of robots, parallel manipulators, actuators and their control, linkage and industrial manipulators, innovative mechanisms/robots and their applications, among others. The book can be used by researchers and engineers in the relevant areas of mechanisms, machines and robotics.

An essential undergraduate textbook on algebra, topology, and calculus An Introduction to Analysis is an essential primer on basic results in algebra, topology, and calculus for undergraduate students considering advanced degrees in mathematics. Ideal for use in a one-year course, this unique textbook also introduces students to rigorous proofs and formal mathematical writing--skills they need to excel. With a range of problems throughout, An Introduction to Analysis treats n-dimensional calculus from the beginning—differentiation, the Riemann integral, series, and differential forms and Stokes's theorem—enabling students who are serious about mathematics to progress quickly to more challenging topics. The book discusses basic material on point set topology, such as normed and metric spaces, topological spaces, compact sets, and the Baire category theorem. It covers linear algebra as well, including vector spaces, linear mappings, Jordan normal form, bilinear mappings, and normal mappings. Proven in the classroom, An Introduction to Analysis is the first textbook to bring these topics together in one easy-to-use and comprehensive volume. Provides a rigorous introduction to calculus in one and several variables Introduces students to basic topology Covers topics in linear algebra, including matrices, determinants, Jordan normal form, and bilinear and normal mappings Discusses differential forms and Stokes's theorem in n dimensions Also covers the Riemann integral, integrability, improper integrals, and series expansions

This is the first volume of a modern introduction to quantum field theory which addresses both mathematicians and physicists, at levels ranging from advanced undergraduate students to professional scientists. The book bridges the acknowledged gap between the different languages used by mathematicians and physicists. For students of mathematics the author shows that detailed knowledge of the physical background helps to motivate the mathematical subjects and to discover interesting interrelationships between quite different mathematical topics. For students of physics, fairly advanced mathematics is presented, which goes beyond the usual curriculum in physics.

Computational methods to approximate the solution of differential equations play a crucial role in science, engineering, mathematics, and technology. The key processes that govern the physical world—wave propagation, thermodynamics, fluid flow, solid deformation, electricity and magnetism, quantum mechanics, general relativity, and many more—are described by differential equations. We depend on numerical methods for the ability to simulate, explore, predict, and control systems involving these processes. The finite element exterior calculus, or FEEC, is a powerful new theoretical approach to the design and understanding of numerical methods to solve partial differential equations (PDEs). The methods derived with FEEC preserve crucial geometric and topological structures underlying the equations and are among the most successful examples of structure-preserving methods in numerical PDEs. This volume aims to help numerical analysts master the fundamentals of FEEC, including the geometrical and functional analysis preliminaries, quickly and in one place. It is also accessible to mathematicians and students of mathematics from areas other than numerical analysis who are interested in understanding how techniques from geometry and topology play a role in numerical PDEs.

Complex analysis is a cornerstone of mathematics, making it an essential element of any area of study in graduate mathematics. Schlag's treatment of the subject emphasizes the intuitive geometric underpinnings of elementary complex analysis that naturally lead to the theory of Riemann surfaces. The book begins with an exposition of the basic theory of holomorphic functions of one complex variable. The first two chapters constitute a fairly rapid, but comprehensive course in complex analysis. The third chapter is devoted to the study of harmonic functions on the disk and the half-plane, with an emphasis on the Dirichlet problem. Starting with the fourth chapter, the theory of Riemann surfaces is developed in some detail and with complete rigor. From the beginning, the geometric aspects are emphasized and classical topics such as elliptic functions and elliptic integrals are presented as illustrations of the abstract theory. The special role of compact Riemann surfaces is explained, and their connection with algebraic equations is established. The book concludes with three chapters devoted to three major results: the Hodge decomposition theorem, the Riemann-Roch theorem, and the uniformization theorem. These chapters present the core technical apparatus of Riemann surface theory at this level. This text is intended as a detailed, yet fast-paced intermediate introduction to those parts of the theory of one complex variable that seem most useful in other areas of mathematics, including geometric group theory, dynamics, algebraic geometry, number theory, and functional analysis. More than seventy figures serve to illustrate concepts and ideas, and the many problems at the end of each chapter give the reader ample opportunity for practice and independent study.

There already exist a number of excellent graduate textbooks on the theory of differential forms as well as a handful of very good undergraduate textbooks on multivariable calculus in which this subject is briefly touched upon but not elaborated on enough.The goal of this textbook is to be readable and usable for undergraduates. It is entirely devoted to the subject of differential forms and explores a lot of its important ramifications.In particular, our book provides a detailed and lucid account of a fundamental result in the theory of differential forms which is, as a rule, not touched upon in undergraduate texts: the isomorphism between the ?ech cohomology groups of a differential manifold and its de Rham cohomology groups.

This textbook treats the classical parts of mapping degree theory, with a detailed account of its history traced back to the first half of the 18th century. After a historical first chapter, the remaining four chapters develop the mathematics. An effort is made to use only elementary methods, resulting in a self-contained presentation. Even so, the book arrives at some truly outstanding theorems: the classification of homotopy classes for spheres and the Poincare-Hopf Index Theorem, as well as the proofs of the original formulations by Cauchy, Poincare, and others. Although the mapping degree theory you will discover in this book is a classical subject, the treatment is refreshing for its simple and direct style. The straightforward exposition is accented by the appearance of several uncommon topics: tubular neighborhoods without metrics, differences between class 1 and class 2 mappings, Jordan Separation with neither compactness nor cohomology, explicit constructions of homotopy classes of spheres, and the direct computation of the Hopf invariant of the first Hopf fibration. The book is suitable for a one-semester graduate course. There are 180 exercises and problems of different scope and difficulty.

Manifolds, the higher-dimensional analogs of smooth curves and surfaces, are fundamental objects in modern mathematics. Combining aspects of algebra, topology, and analysis, manifolds have also been applied to classical mechanics, general relativity, and quantum field theory. In this streamlined introduction to the subject, the theory of manifolds is presented with the aim of helping the reader achieve a rapid mastery of the essential topics. By the end of the book the reader should be able to compute, at least for simple spaces, one of the most basic topological invariants of a manifold, its de Rham cohomology. Along the way, the reader acquires the knowledge and skills necessary for further study of geometry and topology. The requisite point-set topology is included in an appendix of twenty pages; other appendices review facts from real analysis and linear algebra. Hints and solutions are provided to many of the exercises and problems. This work may be used as the text for a one-semester graduate or advanced undergraduate course, as well as by students engaged in self-study. Requiring only minimal undergraduate prerequisites, 'Introduction to Manifolds' is also an excellent foundation for Springer's GTM 82, 'Differential Forms in Algebraic Topology'.

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This book constitutes the refereed proceedings of the 8th International Workshop on Hybrid Systems: Computation and Control, HSCC 2005, held in Zurich, Switzerland in March 2005. The 40 revised full papers presented together with 2 invited papers and the abstract of an invited talk were carefully reviewed and selected from 91 submissions. The papers focus on modeling, analysis, and implementation of dynamic and reactive systems involving both discrete and continuous behaviors. Among the topics addressed are tools for analysis and verification, control and optimization, modeling, engineering applications, and emerging directions in programming language support and implementation.

This volume presents the proceedings of the Joint Summer Research Conference on Algebraic K-theory held at the University of Washington in Seattle. High-quality surveys are written by leading experts in the field. Included is an up-to-date account of Voevodsky's proof of the Milnor conjecture relating the Milnor K-theory of fields to Galois cohomology. The book is intended for graduate students and research mathematicians interested in K-theory, algebraic geometry, and number theory.

This book introduces aspects of topology and applications to problems in condensed matter physics. Basic topics in mathematics have been introduced in a form accessible to physicists, and the use of topology in quantum, statistical and solid state physics has been developed with an emphasis on pedagogy. The aim is to bridge the language barrier between physics and mathematics, as well as the different specializations in physics. Pitched at the level of a graduate student of physics, this book does not assume any additional knowledge of mathematics or physics. It is therefore suited for advanced postgraduate students as well. A collection of selected problems will help the reader learn the topics on one's own, and the broad range of topics covered will make the text a valuable resource for practising researchers in the field. The book consists of two parts: one corresponds to developing the necessary mathematics and the other discusses applications to physical problems. The section on mathematics is a quick, but more-or-less complete, review of topology. The focus is on explaining fundamental concepts rather than dwelling on details of proofs while retaining the mathematical flavour. There is an overview chapter at the beginning and a recapitulation chapter on group theory. The physics section starts with an introduction and then goes on to topics in quantum mechanics, statistical mechanics of polymers, knots, and vertex models, solid state physics, exotic excitations such as Dirac quasiparticles, Majorana modes, Abelian and non-Abelian anyons. Quantum spin liquids and quantum information-processing are also covered in some detail.

This book gives an introduction to fiber spaces and differential operators on smooth manifolds. Over the last 20 years, the authors developed an algebraic approach to the subject and they explain in this book why differential calculus on manifolds can be considered as an aspect of commutative algebra. This new approach is based on the fundamental notion of observable which is used by physicists and will further the understanding of the mathematics underlying quantum field theory.

This monograph (in two volumes) deals with non scalar variational problems arising in geometry, as harmonic mappings between Riemannian manifolds and minimal graphs, and in physics, as stable equilibrium configuations in nonlinear elasticity or for liquid crystals. The presentation is selfcontained and accessible to non specialists. Topics are treated as far as possible in an elementary way, illustrating results with simple examples; in principle, chapters and even sections are readable independently of the general context, so that parts can be easily used for graduate courses. Open questions are often mentioned and the final section of each chapter discusses references to the literature and sometimes supplementary results. Finally, a detailed Table of Contents and an extensive Index are of help to consult this monograph

The power that analysis, topology and algebra bring to geometry has revolutionised the way geometers and physicists look at conceptual problems. Some of the key ingredients in this interplay are sheaves, cohomology, Lie groups, connections and differential operators. In Global Calculus, the appropriate formalism for these topics is laid out with numerous examples and applications by one of the experts in differential and algebraic geometry. Ramanan has chosen an uncommon but natural path through the subject. In this almost completely self-contained account, these topics are developed from scratch. The basics of Fourier transforms, Sobolev theory and interior regularity are proved at the same time as symbol calculus, culminating in beautiful results in global analysis, real and complex. Many new perspectives on traditional and modern questions of differential analysis and geometry are the hallmarks of the book. The book is suitable for a first year graduate course on Global Analysis.

An introductory textbook on cohomology and curvature with emphasis on applications.

This book is aimed to provide an introduction to local cohomology which takes cognizance of the breadth of its interactions with other areas of mathematics. It covers topics such as the number of defining equations of algebraic sets, connectedness properties of algebraic sets, connections to sheaf cohomology and to de Rham cohomology, Grobner bases in the commutative setting as well as for D-modules, the Frobenius morphism and characteristic p methods, finiteness properties of local cohomology modules, semigroup rings and polyhedral geometry, and hypergeometric systems arising from semigroups. The book begins with basic notions in geometry, sheaf theory, and homological algebra leading to the definition and basic properties of local cohomology. Then it develops the theory in a number of different directions, and draws connections with topology, geometry, combinatorics, and algorithmic aspects of the subject.

This book contains a series of papers on some of the longstanding research problems of geometry, calculus of variations, and their applications. It is suitable for advanced graduate students, teachers, research mathematicians, and other professionals in mathematics.

"Classroom-tested in a Princeton University honors course, this text offers a unified introduction to advanced calculus. Starting with an abstract treatment of vector spaces and linear transforms, the authors present a corresponding theory of integration, concluding with a series of applications to analytic functions of complex variables. 1959 edition"--

????:Morse theory

This elegant book is sure to become the standard introduction to synthetic differential geometry. It deals with some classical spaces in differential geometry, namely 'prolongation spaces' or neighborhoods of the diagonal. These spaces enable a natural description of some of the basic constructions in local differential geometry and, in fact, form an inviting gateway to differential geometry, and also to some differential-geometric notions that exist in algebraic geometry. The presentation conveys the real strength of this approach to differential geometry. Concepts are clarified, proofs are streamlined, and the focus on infinitesimal spaces motivates the discussion well. Some of the specific differential-geometric theories dealt with are connection theory (notably affine connections), geometric distributions, differential forms, jet bundles, differentiable groupoids, differential operators, Riemannian metrics, and harmonic maps. Ideal for graduate students and researchers wishing to familiarize themselves with the field.

A study of topology and geometry, beginning with a comprehensible account of the extraordinary and rather mysterious impact of mathematical physics, and especially gauge theory, on the study of the geometry and topology of manifolds. The focus of the book is the Yang-Mills-Higgs field and some considerable effort is expended to make clear its origin and significance in physics. Much of the mathematics developed here to study these fields is standard, but the treatment always keeps one eye on the physics and sacrifices generality in favor of clarity. The author brings readers up the level of physics and mathematics needed to conclude with a brief discussion of the Seiberg-Witten invariants. A large number of exercises are included to encourage active participation on the part of the reader.

Bundles, connections, metrics and curvature are the lingua franca of modern differential geometry and theoretical physics. Supplying graduate students in mathematics or theoretical physics with the fundamentals of these objects, this book would suit a one-semester course on the subject of bundles and the associated geometry.

This book is an easy-to-read reference providing a link between partial differential equations (pde), stochastic analysis, and index theory. Most mathematicians working in pde are only vaguely familiar with the powerful ideas of stochastic analysis. On the other hand, the additional intuition which Taira?s book conveys might provide better insight and be helpful for their work. In addition, the book provides a nice compendium for a large variety of facts from differential geometry, functional analysis, pseudodifferential operators, and Markov processes - for quickly looking up a theorem.

????:Differential geometry of curves and surfaces

Algebraic topology is a basic part of modern mathematics, and some knowledge of this area is indispensable for any advanced work relating to geometry, including topology itself, differential geometry, algebraic geometry, and Lie groups. This book provides a detailed treatment of algebraic topology both for teachers of the subject and for advanced graduate students in mathematics either specializing in this area or continuing on to other fields. J. Peter May's approach reflects the enormous internal developments within algebraic topology over the past several decades, most of which are largely unknown to mathematicians in other fields. But he also retains the classical presentations of various topics where appropriate. Most chapters end with problems that further explore and refine the concepts presented. The final four chapters provide sketches of substantial areas of algebraic topology that are normally omitted from introductory texts, and the book concludes with a list of suggested readings for those interested in delving further into the field.

This unique text brings together into a single framework current research in the three areas of discrete calculus, complex networks, and algorithmic content extraction. Many example applications from several fields of computational science are provided.

The winding number is one of the most basic invariants in topology. It measures the number of times a moving point P goes around a fixed point Q, provided that P travels on a path that never goes through Q and that the final position of P is the same as its starting position. This simple idea has far-reaching applications. The reader of this book will learn how the winding number can help us show that every polynomial equation has a root (the fundamental theorem of algebra),guarantee a fair division of three objects in space by a single planar cut (the ham sandwich theorem),explain why every simple closed curve has an inside and an outside (the Jordan curve theorem),relate calculus to curvature and the singularities of vector fields (the Hopf index theorem),allow one to subtract infinity from infinity and get a finite answer (Toeplitz operators),generalize to give a fundamental and beautiful insight into the topology of matrix groups (the Bott periodicity theorem). All these subjects and more are developed starting only from mathematics that is common in final-year undergraduate courses.

Contributions by three authors treat aspects of noncommutative geometry that are related to cyclic homology. The authors give rather complete accounts of cyclic theory from different points of view. The connections between (bivariant) K-theory and cyclic theory via generalized Chern-characters are discussed in detail. Cyclic theory is the natural setting for a variety of general abstract index theorems. A survey of such index theorems is given and the concepts and ideas involved in these theorems are explained.

This text was produced for the second part of a two-part sequence on advanced calculus, whose aim is to provide a firm logical foundation for analysis. The first part treats analysis in one variable, and the text at hand treats analysis in several variables. After a review of topics from one-variable analysis and linear algebra, the text treats in succession multivariable differential calculus, including systems of differential equations, and multivariable integral calculus. It builds on this to develop calculus on surfaces in Euclidean space and also on manifolds. It introduces differential forms and establishes a general Stokes formula. It describes various applications of Stokes formula, from harmonic functions to degree theory. The text then studies the differential geometry of surfaces, including geodesics and curvature, and makes contact with degree theory, via the Gauss–Bonnet theorem. The text also takes up Fourier analysis, and bridges this with results on surfaces, via Fourier analysis on spheres and on compact matrix groups.

Non-scalar variational problems appear in different fields. In geometry, for in stance, we encounter the basic problems of harmonic maps between Riemannian manifolds and of minimal immersions; related questions appear in physics, for example in the classical

theory of a-models. Non linear elasticity is another example in continuum mechanics, while Oseen-Frank theory of liquid crystals and Ginzburg-Landau theory of superconductivity require to treat variational problems in order to model quite complicated phenomena. Typically one is interested in finding energy minimizing representatives in homology or homotopy classes of maps, minimizers with prescribed topological singularities, topological charges, stable deformations i. e. minimizers in classes of diffeomorphisms or extremal fields. In the last two or three decades there has been growing interest, knowledge, and understanding of the general theory for this kind of problems, often referred to as geometric variational problems. Due to the lack of a regularity theory in the non scalar case, in contrast to the scalar one - or in other words to the occurrence of singularities in vector valued minimizers, often related with concentration phenomena for the energy density - and because of the particular relevance of those singularities for the problem being considered the question of singling out a weak formulation, or completely understanding the significance of various weak formulations becames non trivial.
This text presents a graduate-level introduction to differential geometry for mathematics and physics students. The exposition follows the historical development of the concepts of connection and curvature with the goal of explaining the Chern–Weil theory of characteristic classes on a principal bundle. Along the way we encounter some of the high points in the history of differential geometry, for example, Gauss' Theorema Egregium and the Gauss–Bonnet theorem. Exercises throughout the book test the reader’s understanding of the material and sometimes illustrate extensions of the theory. Initially, the prerequisites for the reader include a passing familiarity with manifolds. After the first chapter, it becomes necessary to understand and manipulate differential forms. A knowledge of de Rham cohomology is required for the last third of the text. Prerequisite material is contained in author's text An Introduction to Manifolds, and can be learned in one semester. For the benefit of the reader and to establish common notations, Appendix A recalls the basics of manifold theory. Additionally, in an attempt to make the exposition more self-contained, sections on algebraic constructions such as the tensor product and the exterior power are included. Differential geometry, as its name implies, is the study of geometry using differential calculus. It dates back to Newton and Leibniz in the seventeenth century, but it was not until the nineteenth century, with the work of Gauss on surfaces and Riemann on the curvature tensor, that differential geometry flourished and its modern foundation was laid. Over the past one hundred years, differential geometry has proven indispensable to an understanding of the physical world, in Einstein's general theory of relativity, in the theory of gravitation, in gauge theory, and now in string theory. Differential geometry is also useful in topology, several complex variables, algebraic geometry, complex manifolds, and dynamical systems, among other fields. The field has even found applications to group theory as in Gromov's work and to probability theory as in Diaconis's work. It is not too far-fetched to argue that differential geometry should be in every mathematician's arsenal.
Based on an honors course taught by the author at UC Berkeley, this introduction to undergraduate real analysis gives a different emphasis by stressing the importance of pictures and hard problems. Topics include: a natural construction of the real numbers, four-dimensional visualization, basic point-set topology, function spaces, multivariable calculus via differential forms (leading to a

simple proof of the Brouwer Fixed Point Theorem), and a pictorial treatment of Lebesgue theory. Over 150 detailed illustrations elucidate abstract concepts and salient points in proofs. The exposition is informal and relaxed, with many helpful asides, examples, some jokes, and occasional comments from mathematicians, such as Littlewood, Dieudonné, and Osserman. This book thus succeeds in being more comprehensive, more comprehensible, and more enjoyable, than standard introductions to analysis. New to the second edition of Real Mathematical Analysis is a presentation of Lebesgue integration done almost entirely using the undergraph approach of Burkill. Payoffs include: concise picture proofs of the Monotone and Dominated Convergence Theorems, a one-line/one-picture proof of Fubini's theorem from Cavalieri's Principle, and, in many cases, the ability to see an integral result from measure theory. The presentation includes Vitali's Covering Lemma, density points — which are rarely treated in books at this level — and the almost everywhere differentiability of monotone functions. Several new exercises now join a collection of over 500 exercises that pose interesting challenges and introduce special topics to the student keen on mastering this beautiful subject.
This book is a survey of the theory of formal deformation quantization of Poisson manifolds, in the formalism developed by Kontsevich. It is intended as an educational introduction for mathematical physicists who are dealing with the subject for the first time. The main topics covered are the theory of Poisson manifolds, star products and their classification, deformations of associative algebras and the formality theorem. Readers will also be familiarized with the relevant physical motivations underlying the purely mathematical construction.

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