

Convex Optimization Stephen Boyd

The purpose of this annual series, Applied and Computational Control, Signals, and Circuits, is to keep abreast of the fast-paced developments in computational mathematics and scientific computing and their increasing use by researchers and engineers in control, signals, and circuits. The series is dedicated to fostering effective communication between mathematicians, computer scientists, computational scientists, software engineers, theorists, and practicing engineers. This interdisciplinary scope is meant to blend areas of mathematics (such as linear algebra, operator theory, and certain branches of analysis) and computational mathematics (numerical linear algebra, numerical differential equations, large scale and parallel matrix computations, numerical optimization) with control and systems theory, signal and image processing, and circuit analysis and design. The disciplines mentioned above have long enjoyed a natural synergy. There are distinguished journals in the fields of control and systems theory, as well as signal processing and circuit theory, which publish high quality papers on mathematical and engineering aspects of these areas; however, articles on their computational and applications aspects appear only sporadically. At the same time, there has been tremendous recent growth and development of computational mathematics, scientific computing, and mathematical software, and the resulting sophisticated techniques are being gradually adapted by engineers, software designers, and other scientists to the needs of those applied disciplines.

Convex optimization is widely used, in many fields, but is nearly always constrained to problems solved in a few minutes or seconds, and even then, nearly always with a human in the loop. The advent of parser-solvers has made convex optimization simpler and more accessible, and greatly increased the number of people using convex optimization. Most current applications, however, are for the design of systems or analysis of data. It is possible to use convex optimization for real-time or embedded applications, where the optimization solver is a part of a larger system. Here, the optimization algorithm must find solutions much faster than a generic solver, and often has a hard, real-time deadline. Use in embedded applications additionally means that the solver cannot fail, and must be robust even in the presence of relatively poor quality data. For ease of embedding, the solver should be simple, and have minimal dependencies on external libraries. Convex optimization has been successfully applied in such settings in the past. However, they have usually necessitated a custom, hand-written solver. This requires significant time and expertise, and has been a major factor preventing the adoption of convex optimization in embedded applications. This work describes the implementation and use of a prototype code generator for convex optimization, CVXGEN, that creates high-speed solvers automatically. Using the principles of disciplined convex programming, CVXGEN allows the user to describe an optimization problem in a convenient,

method is a standard tool for solving unconstrained smooth optimization problems of modest size, proximal algorithms can be viewed as an analogous tool for nonsmooth, constrained, large-scale, or distributed versions of these problems. They are very generally applicable, but are especially well-suited to problems of substantial recent interest involving large or high-dimensional datasets. Proximal methods sit at a higher level of abstraction than classical algorithms like Newton's method: the base operation is evaluating the proximal operator of a function, which itself involves solving a small convex optimization problem. These subproblems, which generalize the problem of projecting a point onto a convex set, often admit closed-form solutions or can be solved very quickly with standard or simple specialized methods. Proximal Algorithms discusses different interpretations of proximal operators and algorithms, looks at their connections to many other topics in optimization and applied mathematics, surveys some popular algorithms, and provides a large number of examples of proximal operators that commonly arise in practice.

Convex Analysis is an emerging calculus of inequalities while Convex Optimization is its application. Analysis is the domain of the mathematician while Optimization belongs to the engineer. In layman's terms, the mathematical science of Optimization is a study of how to make good choices when confronted with conflicting requirements and demands. The qualifier Convex means: when an optimal solution is found, then it is guaranteed to be a best solution; there is no better choice. As any convex optimization problem has geometric interpretation, this book is about convex geometry (with particular attention to distance geometry) and nonconvex, combinatorial, and geometrical problems that can be relaxed or transformed into convexity. A virtual flood of new applications follows by epiphany that many problems, presumed nonconvex, can be so transformed. This is a BLACK & WHITE paperback. A hardcover with full color interior, as originally conceived, is available at lulu.com/spotlight/dattorro

The study of Euclidean distance matrices (EDMs) fundamentally asks what can be known geometrically given only distance information between points in Euclidean space. Each point may represent simply location or, abstractly, any entity expressible as a vector in finite-dimensional Euclidean space. The answer to the question posed is that very much can be known about the points; the mathematics of this combined study of geometry and optimization is rich and deep. Throughout we cite beacons of historical accomplishment. The application of EDMs has already proven invaluable in discerning biological molecular conformation. The emerging practice of localization in wireless sensor networks, the global positioning system (GPS), and distance-based pattern recognition will certainly simplify and benefit from this theory. We study the pervasive convex Euclidean bodies and their various representations. In particular, we make convex polyhedra, cones, and dual cones more visceral through illustration, and we study the geometric relation of polyhedral cones to nonorthogonal bases biorthogonal expansion. We explain conversion between halfspace- and vertex-descriptions of

convex cones, we provide formulae for determining dual cones, and we show how classic alternative systems of linear inequalities or linear matrix inequalities and optimality conditions can be explained by generalized inequalities in terms of convex cones and their duals. The conic analogue to linear independence, called conic independence, is introduced as a new tool in the study of classical cone theory; the logical next step in the progression: linear, affine, conic. Any convex optimization problem has geometric interpretation. This is a powerful attraction: the ability to visualize geometry of an optimization problem. We provide tools to make visualization easier. The concept of faces, extreme points, and extreme directions of convex Euclidean bodies is explained here, crucial to understanding convex optimization. The convex cone of positive semidefinite matrices, in particular, is studied in depth. We mathematically interpret, for example, its inverse image under affine transformation, and we explain how higher-rank subsets of its boundary united with its interior are convex. The Chapter on "Geometry of convex functions", observes analogies between convex sets and functions: The set of all vector-valued convex functions is a closed convex cone. Included among the examples in this chapter, we show how the real affine function relates to convex functions as the hyperplane relates to convex sets. Here, also, pertinent results for multidimensional convex functions are presented that are largely ignored in the literature; tricks and tips for determining their convexity and discerning their geometry, particularly with regard to matrix calculus which remains largely unsystematized when compared with the traditional practice of ordinary calculus. Consequently, we collect some results of matrix differentiation in the appendices. The Euclidean distance matrix (EDM) is studied, its properties and relationship to both positive semidefinite and Gram matrices. We relate the EDM to the four classical axioms of the Euclidean metric; thereby, observing the existence of an infinity of axioms of the Euclidean metric beyond the triangle inequality. We proceed by deriving the fifth Euclidean axiom and then explain why furthering this endeavor is inefficient because the ensuing criteria (while describing polyhedra) grow linearly in complexity and number. Some geometrical problems solvable via EDMs, EDM problems posed as convex optimization, and methods of solution are presented; \eg, we generate a recognizable isotonic map of the United States using only comparative distance information (no distance information, only distance inequalities). We offer a new proof of the classic Schoenberg criterion, that determines whether a candidate matrix is an EDM. Our proof relies on fundamental geometry; assuming, any EDM must correspond to a list of points contained in some polyhedron (possibly at its vertices) and vice versa. It is not widely known that the Schoenberg criterion implies nonnegativity of the EDM entries; proved here. We characterize the eigenvalues of an EDM matrix and then devise a polyhedral cone required for determining membership of a candidate matrix (in Cayley-Menger form) to the convex cone of Euclidean distance matrices (EDM cone); \ie, a candidate is an EDM if and only if its eigenspectrum belongs to a spectral cone for EDM^N . We will see spectral cones

are not unique. In the chapter "EDM cone", we explain the geometric relationship between the EDM cone, two positive semidefinite cones, and the ellipsope. We illustrate geometric requirements, in particular, for projection of a candidate matrix on a positive semidefinite cone that establish its membership to the EDM cone. The faces of the EDM cone are described, but still open is the question whether all its faces are exposed as they are for the positive semidefinite cone. The classic Schoenberg criterion, relating EDM and positive semidefinite cones, is revealed to be a discretized membership relation (a generalized inequality, a new Farkas-like lemma) between the EDM cone and its ordinary dual. A matrix criterion for membership to the dual EDM cone is derived that is simpler than the Schoenberg criterion. We derive a new concise expression for the EDM cone and its dual involving two subspaces and a positive semidefinite cone. "Semidefinite programming" is reviewed with particular attention to optimality conditions of prototypical primal and dual conic programs, their interplay, and the perturbation method of rank reduction of optimal solutions (extant but not well-known). We show how to solve a ubiquitous platonic combinatorial optimization problem from linear algebra (the optimal Boolean solution x to $Ax=b$) via semidefinite program relaxation. A three-dimensional polyhedral analogue for the positive semidefinite cone of 3×3 symmetric matrices is introduced; a tool for visualizing in 6 dimensions. In "EDM proximity" we explore methods of solution to a few fundamental and prevalent Euclidean distance matrix proximity problems; the problem of finding that Euclidean distance matrix closest to a given matrix in the Euclidean sense. We pay particular attention to the problem when compounded with rank minimization. We offer a new geometrical proof of a famous result discovered by Eckart & Young in 1936 regarding Euclidean projection of a point on a subset of the positive semidefinite cone comprising all positive semidefinite matrices having rank not exceeding a prescribed limit ρ . We explain how this problem is transformed to a convex optimization for any rank ρ .

This book is derived from lecture notes for a course on Fourier analysis for engineering and science students at the advanced undergraduate or beginning graduate level. Beyond teaching specific topics and techniques—all of which are important in many areas of engineering and science—the author's goal is to help engineering and science students cultivate more advanced mathematical know-how and increase confidence in learning and using mathematics, as well as appreciate the coherence of the subject. He promises the readers a little magic on every page. The section headings are all recognizable to mathematicians, but the arrangement and emphasis are directed toward students from other disciplines. The material also serves as a foundation for advanced courses in signal processing and imaging. There are over 200 problems, many of which are oriented to applications, and a number use standard software. An unusual feature for courses meant for engineers is a more detailed and accessible treatment of distributions and the generalized Fourier transform. There is also

