Control Theory For Partial Differential Equations Volume 1 Abstract Parabolic Systems Continuous And Approximation Theories Encyclopedia Of Mathematics And Its Applications

"Optimal control theory is concerned with finding control functions that minimize cost functions for systems described by differential equations. The methods have found widespread applications in aeronautics, mechanical engineering, the life sciences, and many other disciplines. This book focuses on optimal control problems where the state equation is an elliptic or parabolic partial differential equation. Included are topics such as the existence of optimal solutions, necessary optimality conditions and adjoint equations, second-order sufficient conditions, and main principles of selected numerical techniques. It also contains a survey on the Karush-Kuhn-Tucker theory of nonlinear programming in Banach spaces. The exposition begins with control problems with linear equations, quadratic cost functions and control constraints. To make the book self-contained, basic facts on weak solutions of elliptic and parabolic equations are introduced. Principles of functional analysis are introduced and explained as they are needed. Many simple examples illustrate the theory and its hidden difficulties. This start to the book makes it fairly self-contained and suitable for advanced undergraduates or beginning graduate students. Advanced control problems for nonlinear partial differential equations are also discussed. As prerequisites, results on boundedness and continuity of solutions to semilinear elliptic and parabolic equations are addressed. These topics are not yet readily available in books on PDEs, making the exposition also interesting for researchers. Alongside the main theme of the analysis of problems of optimal control, Tr'oltzsch also discusses numerical techniques. The exposition is confined to brief introductions into the basic ideas in order to give the reader an impression of how the theory can be realized numerically. After reading this book, the reader will be familiar with the main principles of the numerical analysis of PDE-constrained optimization."--Publisher's description.

Control Theory of Partial Differential EquationsTrends in Control Theory and Partial Differential EquationsSpringer

This book develops a methodology for designing feedback control laws for dynamic traffic assignment (DTA) exploiting the introduction of new sensing and information-dissemination technologies to facilitate the introduction of real-time traffic management in intelligent transportation systems. Three methods of modeling the traffic system are discussed: partial differential equations representing a distributed-parameter setting; continuous-time ordinary differential equations (ODEs) representing a continuous-time lumped-parameter setting; and discreet-time ODEs representing a discrete-time lumped-parameter setting. Feedback control formulations for reaching road-user-equilibrium are presented for each setting and advantages and disadvantage of using each are addressed. The closed-loop methods described are proposed expressly to avoid the counter-productive shifting of bottlenecks from one route to another because of driver over-reaction to routing information. The second edition of Feedback Control Theory for Dynamic Traffic Assignment has been thoroughly updated with completely new chapters: a review of the DTA problem and emphasizing real-time-feedback-based problems; an up-to-date presentation of pertinent traffic-flow theory; and a treatment of the mathematical solution to the traffic dynamics. Techinques accounting for the importance of entropy are further new inclusions at various points in the text. Researchers working in traffic control will find the theoretical material presented a sound basis for further research; the continual reference to applications will help professionals working in highway administration and engineering with the increasingly important task of maintaining and smoothing traffic flow; the extensive use of end-of-chapter exercises will help the graduate student and those new to the field to extend their knowledge.

The field of control theory in PDEs has broadened considerably as more realistic models have been introduced and investigated. This book presents a broad range of recent developments, new discoveries, and mathematical tools in the field. The authors discuss topics such as elasticity, thermo-elasticity, aero-elasticity, interactions between fluids a

Focusing on research surrounding aspects of insufficiently studied problems of estimation and optimal control of random fields, this book exposes some important aspects of those fields for systems modeled by stochastic partial differential equations. It contains many results of interest to specialists in both the theory of random fields and optimal control theory who use modern mathematical tools for resolving specific applied problems, and presents research that has not previously been covered. More generally, this book is intended for scientists, graduate, and post-graduates specializing in probability theory and mathematical statistics. The models presented describe many processes in turbulence theory, fluid mechanics, hydrology, astronomy, and meteorology, and are widely used in pattern recognition theory and parameter identification of stochastic systems. Therefore, this book may also be useful to applied mathematicians who use probability and statistical methods in the selection of useful signals subject to noise, hypothesis distinguishing, distributed parameter systems optimal control, and more. Material presented in this monograph can be used for education courses on the estimation and control theory of random fields.

?This book aims to further develop the theory of stochastic functional inclusions and their applications for describing the solutions of the initial and boundary value problems for partial differential inclusions. The self-contained volume is designed to introduce the reader in a systematic fashion, to new methods of the stochastic optimal control theory from the very beginning. The exposition contains detailed proofs and uses new and original methods to characterize the properties of stochastic functional inclusions that, up to the present time, have only been published recently by the author. The work is divided into seven chapters, with the first two acting as an introduction, containing selected material dealing with point- and set-valued stochastic processes, and the final two devoted to applications and optimal control problems. The book presents recent and pressing issues in stochastic processes, control, differential games, optimization and their application in finance, manufacturing, queueing networks, and climate control. Written by an award-winning author in the field of stochastic differential inclusions and their application to control theory, This book is intended for students and researchers in mathematics and applications; particularly those studying optimal control theory. It is also highly relevant for students of economics and engineering. The book can also be used as a reference on stochastic differential inclusions. Knowledge of select topics in analysis and probability theory are required.

The book provides a quick overview of a wide range of active research areas in partial differential equations. The book can serve as a useful source of information to mathematicians, scientists and engineers. The volume contains contributions from authors from a large variety of countries on different aspects of partial differential equations, such as evolution equations and estimates for their solutions, control theory, inverse problems, nonlinear equations, elliptic theory on singular domains, numerical approaches.

In the development of optimal control, the complexity of the systems to which it is applied has increased significantly, becoming an issue in scientific computing. In order to carry out model-reduction on these systems, the authors of this work have developed a method based on asymptotic analysis. Moving from abstract explanations to examples and applications with a focus on structural network problems, they aim at combining techniques of homogenization and approximation. Optimal Control Problems for Partial Differential Equations on Reticulated Domains is an excellent reference tool for graduate students, researchers, and practitioners in mathematics and areas of engineering involving reticulated domains.

Treats optimal problems for systems described by ODEs and PDEs, using an approach that unifies finite and infinite dimensional nonlinear programming.

Originally published in 2000, this is the second volume of a comprehensive two-volume treatment of quadratic optimal control theory for partial differential equations over a finite or infinite time horizon, and related differential (integral) and algebraic Riccati equations. Both continuous theory and numerical approximation theory are included. The authors use an abstract space, operator theoretic approach, which is based on semigroups methods, and which unifies across a few basic classes of evolution. The various abstract frameworks are motivated by, and ultimately directed to, partial differential equations with boundary/point control. Volume 2 is focused on the optimal control problem over a finite time interval for hyperbolic dynamical systems. A few abstract models are considered, each motivated by a particular canonical hyperbolic dynamics. It presents numerous fascinating results. These volumes will appeal to graduate students and researchers in pure and applied mathematics and theoretical engineering with an interest in optimal control problems.

This volume contains the contributions of participants of the conference "Optimal Control of Partial Differential Equations" which, under the chairmanship of the editors, took place at the Mathematisches Forschungsinstitut Oberwolfach from May 18 to May 24, 1986. The great variety of topics covered by the contributions strongly indicates that also in the future it will be impossible to develop a unifying control theory of partial differential equations. On the other hand, there is a strong tendency to treat prob lems which are directly connected to practical applications. So this volume contains real-world applications like optimal cooling laws for the production of rolled steel or concrete solutions for the problem of optimal shape design in mechanics and hydrody namics. Another main topic is the construction of numerical methods. This includes applications of the finite element method as well as of Quasi-Newton-methods to con strained and unconstrained control problems. Also, very complex problems arising in the theory of free boundary value problems are treated.]~inally, some contribu tions show how practical problems stimulate the further development of the theory; in particular, this is the case for fields like suboptimal control, necessary optimality conditions and sensitivity analysis. As usual, the lectures and stimulating discussions took place in the pleasant at mosphere of the Mathematisches Forschungsinstitut Oberwolfach. Special thanks of the participants are returned to the Director as well as to the staff of the institute.

This book presents novel results by participants of the conference “Control theory of infinite-dimensional systems” that took place in January 2018 at the FernUniversität in Hagen. Topics include well-posedness, controllability, optimal control problems as well as stability of linear and nonlinear systems, and are covered by world-leading experts in these areas. A distinguishing feature of the contributions in this volume is the particular combination of researchers from different fields in mathematics working in an interdisciplinary fashion on joint projects in mathematical system theory. More explicitly, the fields of partial differential equations, semigroup theory, mathematical physics, graph and network theory as well as numerical analysis are all well-represented.

First of a two-volume treatise on deterministic control systems modeled by multi-dimensional partial differential equations, originally published in 2000.

1. The development of a theory of optimal control (deterministic) requires the following initial data: (i) a control u belonging to some set illi ad (the set of 'admissible controls') which is at our disposition, (ii) for a given control u, the state y(u) of the system which is to be controlled is given by the solution of an equation (*) Ay(u)=given function ofu where A is an operator (assumed known) which specifies the system to be controlled (A is the 'model' of the system), (iii) the observation z(u) which is a function of y(u) (assumed to be known exactly; we consider only deterministic problems in this book), (iv) the "cost function" J(u) ("economic function") which is defined in terms of a numerical function z-+

Ordinary differential control thPory (the classical theory) studies input/output re lations defined by systems of ordinary differential equations (ODE). The various con cepts that can be introduced (controllability, observability, invertibility, etc.) must be tested on formal objects (matrices, vector fields, etc.) by means of formal operations (multiplication, bracket, rank, etc.), but without appealing to the explicit integration (search for trajectories, etc.) of the given ODE. Many partial results have been re cently unified by means of new formal methods coming from differential geometry and differential algebra. However, certain problems (invariance, equivalence, linearization, etc.) naturally lead to systems of partial differential equations (PDE). More generally, partial differential control theory studies input/output relations defined by systems of PDE (mechanics, thermodynamics, hydrodynamics, plasma physics, robotics, etc.). One of the aims of this book is to extend the preceding con cepts to this new situation, where, of course, functional analysis and/or a dynamical system approach cannot be used. A link will be exhibited between this domain of applied mathematics and the famous 'Backlund problem', existing in the study of solitary waves or solitons. In particular, we shall show how the methods of differ ential elimination presented here will allow us to determine compatibility conditions on input and/or output as a better understanding of the foundations of control the ory. At the same time we shall unify differential geometry and differential algebra in a new framework, called differential algebraic geometry.

Algebraic analysis, that is the algebraic study of systems of partial differential equations by means of module theory and homological algebra, was pioneered around 1970 by M. Kashiwara, B. Malgrange, and V.P. Palamodov. The theory of differential modules, namely modules over a noncommutative ring of differential operators, is a fashionable subject of research today. However, despite its fundamental importance in mathematics, it can only be found in specialist books and papers, and has only been applied in control theory since 1990. This book provides an account of algebraic analysis and its application to control systems defined by partial differential equations. The first volume presents the mathematical tools needed from both commutative algebra, homological algebra, differential geometry and differential algebra. The second volume applies these new methods in order to study the structural and input/output properties of both linear and nonlinear control systems. Hundreds of explicit examples allow the reader to gain insight and experience in these topics.

This document describes research activity supported under this grant from its inception (June, 1982) to the present. (KR).

Annotation The field of control theory in PDEs has broadened considerably as more realistic models have been introduced and investigated. This book presents a broad range of recent developments, new discoveries, and mathematical tools in the field. The authors discuss topics such as elasticity, thermo-elasticity, aero-elasticity, interactions between fluids and elastic structures, and fluid dynamics and the new challenges that they present. Other control theoretic problems include parabolic systems, dynamical Lame systems, linear and nonlinear hyperbolic equations, and pseudo-differential operators on a manifold.

The articles in this volume reflect a subsequent development after a scientific meeting entitled Carleman Estimates and Control Theory, held in Cartona in September 1999. The 14 research-level articles, written by experts, focus on new results on Carleman estimates and their applications to uniqueness and controlla bility of partial differential equations and systems. The main topics are unique continuation for elliptic PDEs and systems, con trol theory and inverse problems. New results on strong uniqueness for second or higher order operators are explored in detail in several papers. In the area of control theory. the reader will find applications of Carleman estimates to stabiliza tion, observability and exact control for the wave and the SchrOdinger equations. A final paper presents a challenging list of open problems on the topic of control lability of linear and sernilinear heat equations. The papers contain exhaustive and essentially self-contained proofs directly ac cessible to mathematicians, physicists, and graduate students with an elementary background in PDEs. Contributors are L. Aloui, M. Bellassoued, N. Burq, F. Colombini, B. Dehman, C. Grammatico, M. Khenissi, H. Koch, P. Le Borgne, N. Lerner, T. Nishitani. T. Okaji, K.D. Phung, R. Regbaoui, X. Saint Raymond, D. Tataru, and E. Zuazua.

This is the first volume of a comprehensive and up-to-date treatment of quadratic optimal control theory for partial differential equations over a finite or infinite time horizon, and related differential (integral) and algebraic Riccati equations. The authors describe both continuous theory and numerical approximation. They use an abstract space, operator theoretic approach, based on semigroups methods and unifying across a few basic classes of evolution. The various abstract frameworks are motivated by, and ultimately directed to, partial differential equations with boundary/point control. Volume I includes the abstract parabolic theory (continuous theory and numerical approximation theory) for the finite and infinite cases and corresponding PDE illustrations, and presents numerous new results. These volumes will appeal to graduate students and researchers in pure and applied mathematics and theoretical engineering with an interest in optimal control problems.

Progress in Partial Differential Equations is devoted to modern topics in the theory of partial differential equations. It consists of both original articles and survey papers covering a wide scope of research topics in partial differential equations and their applications. The contributors were participants of the 8th ISAAC congress in Moscow in 2011 or are members of the PDE interest group of the ISAAC society. This volume is addressed to graduate students at various levels as well as researchers in partial differential equations and related fields. The readers will find this an excellent resource of both introductory and advanced material. The key topics are: • Linear hyperbolic equations and systems (scattering, symmetrisers) • Non-linear wave models (global existence, decay estimates, blow-up) • Evolution equations (control theory, well-posedness, smoothing) • Elliptic equations (uniqueness, non-uniqueness, positive solutions) • Special models from applications (Kirchhoff equation, Zakharov-Kuznetsov equation, thermoelasticity)

This volume contains selected papers that were presented at the AMS-IMS-SIAM Joint Summer Research Conference on ""Differential Geometric Methods in the Control of Partial Differential Equations"", which was held at the University of Colorado in Boulder in June 1999. The aim of the conference was to explore the infusion of differential-geometric methods into the analysis of control theory of partial differential equations, particularly in the challenging case of variable coefficients, where the physical characteristics of the medium vary from point to point. While a mutually profitable link has been long established, for at least 30 years, between differential geometry and control of ordinary differential equations, a comparable relationship between differential geometry and control of partial differential equations (PDEs) is a new and promising topic.Very recent research, just prior to the Colorado conference, supported the expectation that differential geometric methods, when brought to bear on classes of PDE modelling and control problems with variable coefficients, will yield significant mathematical advances. The papers included in this volume - written by specialists in PDEs and control of PDEs as well as by geometers - collectively support the claim that the aims of the conference are being fulfilled. In particular, they endorse the belief that both subjects - differential geometry and control of PDEs - have much to gain by closer interaction with one another. Consequently, further research activities in this area are bound to grow.

This book presents cutting-edge contributions in the areas of control theory and partial differential equations. Over the decades, control theory has had deep and fruitful interactions with the theory of partial differential equations (PDEs). Well-known examples are the study of the generalized solutions of Hamilton-Jacobi-Bellman equations arising in deterministic and stochastic optimal control and the development of modern analytical tools to study the controllability of infinite dimensional systems governed by PDEs. In the present volume, leading experts provide an up-to-date overview of the connections between these two vast fields of mathematics. Topics addressed include regularity of the value function associated to finite dimensional control systems, controllability and observability for PDEs, and asymptotic analysis of multiagent systems. The book will be of interest for both researchers and graduate students working in these areas.

This is the first book to systematically present control theory for stochastic distributed parameter systems, a comparatively new branch of mathematical control theory. The new phenomena and difficulties arising in the study of controllability and optimal control problems for this type of system are explained in detail. Interestingly enough, one has to develop new mathematical tools to solve some problems in this field, such as the global Carleman estimate for stochastic partial differential equations and the stochastic transposition method for backward stochastic evolution equations. In a certain sense, the stochastic distributed parameter control system is the most general control system in the context of classical physics. Accordingly, studying this field may also yield valuable insights into quantum control systems. A basic grasp of functional analysis, partial differential equations, and control theory for deterministic systems is the only prerequisite for reading this book.

Nonlinear Optimal Control Theory presents a deep, wide-ranging introduction to the mathematical theory of the optimal control of processes governed by ordinary differential equations and certain types of differential equations with memory. Many examples illustrate the mathematical issues that need to be addressed when using optimal control techniques in diverse areas. Drawing on classroom-tested material from Purdue University and North Carolina State University, the book gives a unified account of bounded state problems governed by ordinary, integrodifferential, and delay systems. It also discusses Hamilton-Jacobi theory. By providing a sufficient and rigorous treatment of finite dimensional control problems, the book equips readers with the foundation to deal with other types of control problems, such as those governed by stochastic differential equations, partial differential equations, and differential games.

This textbook presents, in a mathematically precise manner, a unified introduction to deterministic control theory. With the exception of a few more advanced concepts required for the final part of the book, the presentation requires only a knowledge of basic facts from linear algebra, differential equations, and calculus. In addition to classical concepts and ideas, the author covers the stabilization of nonlinear systems using topological methods, realization theory for nonlinear systems, impulsive control and positive systems, the control of rigid bodies, the stabilization of infinite dimensional systems, and the solution of minimum energy problems. This second edition includes new chapters that introduce a variety of topics, such as controllability with vanishing energy, boundary control systems, and delayed systems. With additional proofs, theorems, results, and a substantially larger index, this new edition will be an invaluable resource for students and researchers of control theory. Mathematical Control Theory: An Introduction will be ideal for a beginning graduate course in mathematical control theory, or for self-study by professionals needing a complete picture of the mathematical theory that underlies the applications of control theory. From reviews of the first edition: At last! We did need an introductory textbook on control which can be read, understood, and enjoyed by anyone. Gian-Carlo Rota, The Bulletin of Mathematics Books It covers a remarkable number of topics...The exposition is excellent, and the book is a joy to read. A novel one-semester course covering both linear and nonlinear systems could be given...The book is an excellent one for introducing a mathematician to control theory. Bulletin of the AMS Indeed, for mathematicians who look for the basic ideas or a general picture about the main branches of control theory, I believe this book can provide an excellent bridge to this area. IEEE Control Systems Magazine

The study of optimal control problems for distributed parameter systems is intimately connected to the study of partial differential equations and/or integral equations involving two or more independent variables. The primary purpose of the dissertation is to investigate the application of optimal control theory to systems described by partial differential equations. The intent is not to advance the theory of partial differential equations per se. Thus all considerations will be restricted to the more familiar equations of the type which often occur in mathematical physics. Specifically, the distributed parameter systems under consideration are represented by a set of field equations.