Application Of Laplace Transform In Mechanical Engineering

The theory of Laplace transformation is an important part of the mathematical background required for engineers, physicists and mathematicians. Laplace transformation methods provide easy and effective techniques for solving many problems arising in various fields of science and engineering, especially for solving differential equations. What the Laplace transformation does in the field of differential equations, the z-transformation achieves for difference equations. The two theories are parallel and have many analogies. Laplace and z transformations are also referred to as operational calculus, but this notion is also used in a more restricted sense to denote the operational calculus of Mikusinski. This book does not use the operational calculus of Mikusinski, whose approach is based on abstract algebra and is not readily accessible to engineers and scientists. The symbolic computation capability of Mathematica can now be used in favor of the Laplace and z-transformations. The first version of the Mathematica Package LaplaceAndzTransforrns developed by the author appeared ten years ago. The Package computes not only Laplace and z-transforms but also includes many routines from various domains of applications. Upon loading the Package, about one hundred and fifty new commands are added to the built-in commands of Mathematica. The code is placed in front of the already built-in code of Laplace and z-transformations of Mathematica so that built-in functions not covered by the Package remain available. The Package substantially enhances the Laplace and z-transformation facilities of Mathematica. The book is mainly designed for readers working in the field of applications.

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The application of the Laplace transformation to the solution of the lateral and longitudinal stability equations is presented. The expressions for the time history of the motion in response to a sinusoidal control motion are derived for the general case in which the initial conditions, initial displacements and initial velocities, are assumed different from zero. Some illustrative examples of the application of the Laplace transform to ordinary linear differential equations with constant coefficients and a number example of a specific problem are presented in appendixes.

There is a lot of literature devoted to operational calculus, which includes the analysis of properties and rules of integral transformations and illustrates their usefulness in different fields of applied mathematics, engineering and natural sciences. The integral transform technique is one of most useful tools of applied mathematics employed in many branches of science and engineering. Typical applications include the design and analysis of transient and steady-state configurations of linear systems in electrical, mechanical and control engineering, and heat transfer, diffusion, waves, vibrations and fluid motion problems. The Laplace transformation receives special attention in literature because of its importance in various applications and therefore is considered as a standard technique in solving linear differential equations. For this reason, this book is centered on the Laplace transformation. (Imprint: Nova)

This is a revised edition of the chapter on Laplace Transforms, which was published few years ago in Part II of My Personal Study Notes in advanced mathematics. In this edition, I typed the cursive scripts of the personal notes, edited the typographic errors, but most of all reproduced all the calculations and graphics in a modern style of representation. The book is organized into six chapters equally distributed to address: (1) The theory of Laplace transformations and inverse transformations of elementary functions, supported by solved examples and exercises with given answers; (2) Transformation of more complex functions from elementary transformation; (3) Practical applications of Laplace transformation to equations of motion of material bodies and deflection, stress, and strain of elastic beams; (4) Solving equations of state of motion of bodies under inertial and gravitational forces. (5) Solving heat flow equations through various geometrical bodies; and (6) Solving partial differential equations by the operational algebraic properties of transforming and inverse transforming of partial differential equations. During the editing process, I added plenty of comments of the underlying meaning of the arcane equations such that the reader could discern the practical weight of each mathematical formula. In a way, I attempted to convey a personal sense and feeling on the significance and philosophy of devising a mathematical equation that transcends into real-life emulation. The reader will find this edition dense with graphic illustrations that should spare the reader the trouble of searching other references in order to infer any missing steps. In my view, detailed graphic illustrations could soothe the harshness of arcane mathematical jargon, as well as expose the merits of the assumption contemplated in the formulation. In lieu of offering a dense textbook on Laplace Transforms, I opted to stick to my personal notes that give the memorable zest of a subject that could easily remembered when not frequently used. Brief Outline of Contents: CHAPTER 1. THE LAPLACE TRANSFORMATION AND INVERSE TRANSFORMATION 1.1. Integral transforms 1.2. Some elementary Laplace transforms 1.3. The Laplace transformation of the sum of two functions 1.4. Sectionally or piecewise continuous functions 1.5. Functions of exponential order 1.7. Null functions 1.8. Inverse Laplace transforms 1.10. Laplace transforms of derivatives 1.11. Laplace transforms of integrals 1.12. The first shift theorem of multiplying the object function by eat 1.15. Determination of the inverse Laplace transforms by the aid of partial fractions 1.16. Laplace's solution of linear differential equations with constant coefficients CHAPTER 2. GENERAL THEOREMS ON THE LAPLACE TRANSFORMATION 2.1. The unit step function 2.2. The second translation or shifting property 2.4. The unit impulse function 2.5. The unit doublet 2.7. Initial value theorem 2.8. Final value theorem 2.9. Differentiation of transform 2.11. Integration of transforms 2.12. Transforms of periodic functions 2.13. The product theorem-Convolution 2.15. Power series method for the determination of transforms and inverse transforms 2.16. The error function or probability integral 2.22. The inversion integral CHAPTER 3. ELECTRICAL APPLICATIONS OF THE LAPLACE TRANSFORMATION CHAPTER 4. DYNAMICAL APPLICATIONS OF LAPLACE TRANSFORMS CHAPTER 5. STRUCTURAL APPLICATIONS 5.1. Deflection of beams CHAPTER 6. USING LAPLACE TRANSFORMATION IN SOLVING LINEAR PARTIAL DIFFERENTIAL EQUATIONS 6.1. Transverse vibrations of a stretched string under gravity 6.2. Longitudinal vibrations of bars 6.3. Partial differential equations of transmission lines 6.4. Conduction of heat 6.5. Exercise on using Laplace Transformation in solving Linear Partial Differential Equations

Acclaimed text on essential engineering mathematics covers theory of complex variables, Cauchy-Riemann equations, conformal mapping, and multivalued functions, plus Fourier and Laplace transform theory, with applications to engineering, including integrals, linear integrodifferential equations, Z-transform, more. Ideal for home study as well as graduate engineering courses, this volume includes many problems.

Laplace Transforms for Electronic Engineers, Second (Revised) Edition details the theoretical concepts and practical application of Laplace transformation in the context of electrical engineering. The title is comprised of 10 chapters that cover the whole spectrum of Laplace transform theory that includes advancement, concepts, methods, logic, and application. The book first covers the functions of a complex variable, and then proceeds to tackling the Fourier series and integral, the Laplace transformation, and the inverse Laplace transformation. The next chapter details the Laplace transform theorems. The subsequent chapters talk about the various applications of the Laplace transform theories, such as network analysis, transforms of special waveshapes and pulses, electronic filters, and other specialized applications. The text will be of great interest to electrical engineers and technicians.

Introduction to the Theory and Application of the Laplace TransformationSpringer Science & Business Media

This introduction to Laplace transforms and Fourier series is aimed at second year students in applied mathematics. It is unusual in treating Laplace transforms at a relatively simple level with many examples. Mathematics students do not usually meet this material until later in their degree course but applied mathematicians and engineers need an early introduction. Suitable as a course text, it will also be of interest to physicists and engineers as supplementary material.

In anglo-american literature there exist numerous books, devoted to the application of the Laplace transformation in technical domains such as electrotechnics, mechanics etc. Chiefly, they treat problems which, in mathematical language, are governed by ordi nary and partial differential equations, in various physically dressed forms. The theoretical foundations of the Laplace transformation are presented usually only in a simplified manner, presuming special properties with respect to the transformed func tions, which allow easy proofs. By contrast, the present book intends principally to develop those parts of the theory of the Laplace transformation, which are needed by mathematicians, physicists a,nd engineers in their daily routine work, but in complete generality and with detailed, exact proofs. The applications to other mathematical domains and to technical prob lems are inserted, when the theory is adequately· developed to present the tools necessary for their treatment. Since the book proceeds, not in a rigorously systematic manner, but rather from easier to more difficult topics, it is suited to be read from the beginning as a textbook, when one wishes to familiarize oneself for the first time with the Laplace transforma tion. For those who are interested only in particular details, all results are specified in "Theorems" with explicitly formulated assumptions and

assertions. Chapters 1-14 treat the question of convergence and the mapping properties of the Laplace transformation. The interpretation of the transformation as the mapping of one function space to another (original and image functions) constitutes the dom inating idea of all subsequent considerations.
A 2003 textbook on Fourier and Laplace transforms for undergraduate and graduate students.
Keeping the style, content, and focus that made the first edition a bestseller, Integral Transforms and their Applications, Second Edition stresses the development of analytical skills rather than the importance of more abstract formulation. The authors provide a working knowledge of the analytical methods required in pure and applied mathematics, physics, and engineering. The second edition includes many new applications, exercises, comments, and observations with some sections entirely rewritten. It contains more than 500 worked examples and exercises with answers as well as hints to selected exercises. The most significant changes in the second edition include: New chapters on fractional calculus and its applications to ordinary and partial differential equations, wavelets and wavelet transformations, and Radon transform Revised chapter on Fourier transforms, including new sections on Fourier transforms of generalized functions, Poissons summation formula, Gibbs phenomenon, and Heisenbergs uncertainty principle A wide variety of applications has been selected from areas of ordinary and partial differential equations, integral equations, fluid mechanics and elasticity, mathematical statistics, fractional ordinary and partial differential equations, and special functions A broad spectrum of exercises at the end of each chapter further develops analytical skills in the theory and applications of transform methods and a deeper insight into the subject A systematic mathematical treatment of the theory and method of integral transforms, the book provides a clear understanding of the subject and its varied applications in mathematics, applied mathematics, physical sciences, and engineering.
The purpose of this book is to give an introduction to the Laplace transform on the undergraduate level. The material is drawn from notes for a course taught by the author at the Milwaukee School of Engineering. Based on classroom experience, an attempt has been made to (1) keep the proofs short, (2) introduce applications as soon as possible, (3) concentrate on problems that are difficult to handle by the older classical methods, and (4) emphasize periodic phenomena. To make it possible to offer the course early in the curriculum (after differential equations), no knowledge of complex variable theory is assumed. However, since a thorough study of Laplace. transforms requires at least the rudiments of this theory, Chapter 3 includes a brief sketch of complex variables, with many of the details presented in Appendix A. This plan permits an introduction of the complex inversion formula, followed by additional applications. The author has found that a course taught three hours a week for a quarter can be based on the material in Chapters 1, 2, and 5 and the first three sections of Chapter 7. If additional time is available (e.g., four quarter-hours or three semester-hours), the

whole book can be covered easily. The author is indebted to the students at the Milwaukee School of Engineering for their many helpful comments and criticisms. Classic graduate-level exposition covers theory and applications to ordinary and partial differential equations. Includes derivation of Laplace transforms of various functions, Laplace transform for a finite interval, and more. 1948 edition. Special Functions Fourier Transforms Laplace Transforms Chapter 7: SPECIAL FUNCTIONS 1. Gamma Functions 2. Beta Functions 3. Bessel Functions 3.1. Bessel equation for zero order 3.2. Properties of Bessel functions 3.3. Bessel functions of order one (n = 1) 3.4. Relationships between Bessel functions of orders zero and one 3.5. Lammel' s integrals 3.6. Fourier-Bessel expansions of zero order 3.7. Vibration of uniformly stretched membrane 3.8. Application of Bessel functions on conduction of heat 3.9. Modified Bessel function of zero order 3.10. Bessel and Kelvin functions 3.11. Bessel functions of any real order 3.12. Bessel functions of integral order 3.13. Bessel coefficients 3.14. Recurrence formulae 3.15. Bessel function as integrals 3.16. The Bessel functions of order n of the third kind (Hankel functions of order n) 4. Legendre functions 4.1. Alternative definition of Legendre polynomials 4.2. Legendre's recurrence formulae 4.3. Integral properties of Lagendre polynomial 4.4. The associated Legendre functions. 4.5. Applications of Legendre functions 5. Exercises on special functions Chapter 8: Fourier transforms 1. Fourier series and harmonic analysis 2. Fourier theorem 3. Preliminary integrals used in Fourier transforms 4. Determination of the coefficients of the Fourier expansion 5. Examples of Fourier transformations 6. Fourier expansions in cosines only 7. Fourier expansions in sines only 8. Fourier expansions in even harmonics 9. Fourier expansions in odd harmonics 10. Summary of common Fourier transforms 11. Practical Fourier Analysis Chapter 11: Laplace Transforms 1. The Laplace Transformation 2. General Theorems on the Laplace Transformation 2.1. The unit step function 2.2. The second translation or shifting property 2.3. Application of the shift theorem to the solution of difference and differential equations 2.4. The unit impulse function 2.5. The unit doublet 2.6. The behavior of f(s) as s tends to infinity. 2.7. Initial value theorem 2.8. Final value theorem 2.9. Differentiation of transform 2.10. Application of the differentiation of Laplace transform to the solution of linear differential equations with coefficients as polynomials in t. 2.11. Integration of transforms 2.12. Transforms of periodic functions 2.13. The product theorem-Convolution 2.14. Application of the product theorem to the solution of differential and integral equations 2.15. Power series method for the determination of transforms and inverse transforms 2.16. The error function or probability integral 2.17. The sine-integral function Si(t) 2.18. The Cosine -integral function Ci(t) 2.19. The exponential integral function 2.20. Evaluation of definite integrals using the Laplace transformation 2.21. The Heaviside's expansion formulae 2.22. The inversion integral 2.23. Formulae for residues 2.24. Inversion in the case of branch points 2.25. Miscellaneous Examples on Laplace Transform 2.26. Exercises on Laplace Transforms 3.

Electrical Applications of the Laplace Transformation 4. Dynamical Applications of Laplace Transforms 5. Structural Applications 5.1. Deflection of beams 5.2. Exercises on Laplace Transform in practical applications 6. Using Laplace Transformation in solving Linear Partial Differential Equations 6.1. Transverse vibrations of a stretched string under gravity 6.2. Longitudinal vibrations of bars 6.3. Partial differential equations of transmission lines 6.4. Conduction of heat 6.5. Exercise on using Laplace Transformation in solving Linear Partial Differential Equations

Topics include the Laplace transform, the inverse Laplace transform, special functions and properties, applications to ordinary linear differential equations, Fourier transforms, applications to integral and difference equations, applications to boundary value problems, and tables.

One of the first applications of the modern Laplace transform was by Bateman in 1910 who used it to transform Rutherford's equations in his work on radioactive decay. The modeling of complex engineering and physical problems by linear differential equations has made the Laplace transform an indispensable mathematical tool for engineers and scientists. The method of Laplace transform for solving linear differential equations is very popular in the disciplines of electrical engineering, environmental engineering, hydrology, and petroleum engineering. This book presents some applications of Laplace transforms in these disciplines. Algorithms for the numerical inversion of Laplace transform are given, and a computer program in R for the Stehfest algorithm is included.

Linear evolution equations in Banach spaces have seen important developments in the last two decades. This is due to the many different applications in the theory of partial differential equations, probability theory, mathematical physics, and other areas, and also to the development of new techniques. One important technique is given by the Laplace transform. It played an important role in the early development of semigroup theory, as can be seen in the pioneering monograph by Rille and Phillips [HP57]. But many new results and concepts have come from Laplace transform techniques in the last 15 years. In contrast to the classical theory, one particular feature of this method is that functions with values in a Banach space have to be considered. The aim of this book is to present the theory of linear evolution equations in a systematic way by using the methods of vector-valued Laplace transforms. It is simple to describe the basic idea relating these two subjects. Let A be a closed linear operator on a Banach space X. The Cauchy problern defined by A is the initial value problern (t 2 0), (CP) {u'(t) = Au(t) u(O) = x, where x E X is a given initial value. If u is an exponentially bounded, continuous function, then we may consider the Laplace transform 00 u(>.) = 1 e-). . tu(t) dt of u for large real>. .

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The Laplace transform is a wonderful tool for solving ordinary and partial differential equations and has enjoyed much success in this realm. With its success, however, a certain casualness has been bred concerning its application, without much regard for hypotheses and when they are valid. Even proofs of theorems often lack rigor, and dubious mathematical practices are not

uncommon in the literature for students. In the present text, I have tried to bring to the subject a certain amount of mathematical correctness and make it accessible to un dergraduates. Th this end, this text addresses a number of issues that are rarely considered. For instance, when we apply the Laplace trans form method to a linear ordinary differential equation with constant coefficients, any(n) + an-lY(n-l) + · · · + aoy = f(t), why is it justified to take the Laplace transform of both sides of the equation (Theorem A. 6)? Or, in many proofs it is required to take the limit inside an integral. This is always fraught with danger, especially with an improper integral, and not always justified. I have given complete details (sometimes in the Appendix) whenever this procedure is required. IX X Preface Furthermore, it is sometimes desirable to take the Laplace trans form of an infinite series term by term. Again it is shown that this cannot always be done, and specific sufficient conditions are established to justify this operation.
This book is devoted to one of the most critical areas of applied mathematics, namely the Laplace transform technique for linear time invariance systems arising from the fields of electrical and mechanical engineering. It focuses on introducing Laplace transformation and its operating properties, finding inverse Laplace transformation through different methods, and describing transfer function applications for mechanical and electrical networks to develop input and output relationships. It also discusses solutions of initial value problems, the state-variables approach, and the solution of boundary value problems connected with partial differential equations.