Algebra With Galois Theory American Mathematical Society

"This book is designed as a text for the first year of graduate algebra, but it can also serve as a reference since it contains more advanced topics as well. This second edition has a different organization than the first. It begins with a discussion of the cubic and quartic equations, which leads into permutations, group theory, and Galois theory (for finite extensions; infinite Galois theory is discussed later in the book). The study of groups continues with finite abelian groups (finitely generated groups are discussed later, in the context of module theory), Sylow theorems, simplicity of projective unimodular groups, free groups and presentations, and the Nielsen-Schreier theorem (subgroups of free groups are free). The study of commutative rings continues with prime and maximal ideals, unique factorization, noetherian rings, Zorn's lemma and applications, varieties, and Gr'obner bases. Next, noncommutative rings and modules are discussed, treating tensor product, projective, injective, and flat modules, categories, functors, and natural transformations, categorical constructions (including direct and inverse limits), and adjoint functors. Then follow group representations: Wedderburn-Artin theorems, character theory, theorems of Burnside and Frobenius, division rings, Brauer groups, and abelian categories. Advanced linear algebra treats canonical forms for matrices and the structure of modules over PIDs, followed by multilinear algebra. Homology is introduced, first for simplicial complexes, then as derived functors, with applications to Ext, Tor, and cohomology of groups, crossed products, and an introduction to algebraic K-theory. Finally, the author treats localization, Dedekind rings and algebraic number theory, and homological dimensions. The book ends with the proof that regular local rings have unique factorization."--Publisher's description.

This volume offers brief treatises on several mathematical areas and a historical summary of American contributions to mathematics during the Society's first fifty years. To download free chapters of this book, click here.

Survey and research papers in this volume are based on talks given at a workshop held at The Fields Institute for Research in the Mathematical Sciences (Toronto, ON, Canada). It provides an up-to-date account by leading researchers on the many current connections among Galois theories, Hopf algebras, and semiabelian categories.

This book, part of the series Contributions in Mathematical and Computational Sciences, reviews recent developments in the theory of vertex operator algebras (VOAs) and their applications to mathematics and physics. The mathematical theory of VOAs originated from the famous monstrous moonshine conjectures of J.H. Conway and S.P. Norton, which predicted a deep relationship between the characters of the largest simple finite sporadic group, the Monster and the theory of modular forms inspired by the observations of J. MacKay and J. Thompson. The contributions are based on lectures delivered at the 2011 conference on Conformal Field Theory, Automorphic Forms and Related Topics, organized by the editors as part of a special program offered at Heidelberg University that summer under the sponsorship of the Mathematics Center Heidelberg (MATCH).

This book is devoted to the relation between two different concepts of integrability: the complete integrability of complex analytical Hamiltonian systems and the integrability of complex analytical linear differential equations. For linear differential equations, integrability is made precise within the framework of differential Galois theory. The connection of these two integrability notions is given by the variational equation (i.e. linearized equation) along a particular integral curve of the Hamiltonian system. The underlying heuristic idea, which motivated the main results presented in this monograph, is that a necessary condition for the integrability of a Hamiltonian system is the integrability of the variational equation along any of its particular integral curves. This idea led to the algebraic non-integrability criteria for Hamiltonian systems. These criteria can be considered as generalizations of classical non-integrability results by Poincaré and Lyapunov, as well as more recent results by Ziglin and Yoshida. Thus, by means of the differential Galois theory it is not only possible to understand all these approaches in a unified way but also to improve them. Several important applications are also included: homogeneous potentials, Bianchi IX cosmological model, three-body problem, Hénon-Heiles system, etc. The book is based on the original joint research of the author with J.M. Peris, J.P. Ramis and C. Simó, but an effort was made to present these achievements in their logical order rather than their historical one. The necessary background on differential Galois theory and Hamiltonian systems is included, and several new problems and conjectures which open new lines of research are proposed. - - - The book is an excellent introduction to non-integrability methods in Hamiltonian mechanics and brings the reader to the forefront of research in the area. The inclusion of a large number of worked-out examples, many of wide applied interest, is commendable. There are many historical references, and an extensive bibliography. (Mathematical Reviews) For readers already prepared in the two prerequisite subjects [differential Galois theory and Hamiltonian dynamical systems], the author has provided a logically accessible account of a remarkable interaction between differential algebra and dynamics. (Zentralblatt MATH)

This book is the second of two proceedings volumes stemming from the International Conference and Workshop on Valuation Theory held at the University of Saskatchewan (Saskatoon, SK, Canada). It contains the most recent applications of valuation theory to a broad range of mathematical ideas. Valuation theory arose in the early part of the twentieth century in connection with number theory and continues to have many important applications to algebra, geometry, and analysis. The research and survey papers in this volume cover a variety of topics, including Galois theory, the Grunwald-Wang Theorem, algebraic geometry, resolution of singularities, curves over Prufer domains, model theory of valued fields and the Frobenius, Hardy fields, Hensel's Lemma, fixed point theorems, and computations in valued fields. It is suitable for graduate students and research mathematicians interested in algebra, algebraic geometry, number theory, and mathematical logic.

Praise for the Third Edition ". . . an expository masterpiece of the highest didactic value that has gained additional attractivity through the various improvements . . ."—Zentralblatt MATH The Fourth Edition of Introduction to Abstract Algebra continues to provide an accessible approach to the basic structures of abstract algebra: groups, rings, and fields. The book's unique presentation helps readers advance to abstract theory by presenting concrete examples of induction, number theory, integers modulo n, and permutations before the abstract structures are defined. Readers can immediately begin to perform computations using abstract concepts that are developed in greater detail later in the text. The Fourth Edition features important concepts as well as specialized topics, including: The treatment of nilpotent groups, including the Frattini and Fitting subgroups Symmetric polynomials The proof of the fundamental theorem of algebra using symmetric polynomials The proof of Wedderburn's theorem on finite division rings The proof of the Wedderburn-Artin theorem Throughout the book, worked examples and real-world problems illustrate concepts and their applications, facilitating a complete understanding for readers regardless of their background in mathematics. A wealth of computational and theoretical exercises, ranging from basic to complex, allows readers to test their comprehension of the material. In addition, detailed historical notes and biographies of mathematicians provide context for and illuminate the discussion of key topics. A solutions manual is also available for readers who would like access to partial solutions to the book's exercises. Introduction to Abstract Algebra, Fourth Edition is an excellent book for courses on the topic at the upper-undergraduate and beginning-graduate levels. The book also serves as a valuable reference and self-study tool for practitioners in the fields of engineering, computer science, and applied mathematics.

This book, the second of three related volumes on number theory, is the English translation of the original Japanese book. Here, the idea of class field theory, a highlight in algebraic number theory, is first described with many concrete examples. A detailed account of proofs is thoroughly exposited in the final chapter. With this book, the reader can enjoy the beauty of numbers and obtain fundamental knowledge of modern number theory.

The present text was first published in 1947 by the Courant Institute of Mathematical Sciences of New York University. Published under the title Modern Higher Algebra. Galois Theory, it was based on lectures by Emil Artin and written by Albert A. Blank. This volume became one of the most popular in the series of lecture notes published by Courant. Many instructors used the book as a textbook, and it was popular among students as a supplementary text as well as a primary textbook. Because of its popularity, Courant has republished the volume under the new title Algebra with Galois Theory.

This book is the outcome of research initiatives formed during the special ``Research Trimester on Multiple Zeta Values, Multiple Polylogarithms, and Quantum Field Theory'' at the ICMAT (Instituto de Ciencias Matemáticas, Madrid) in 2014. The activity was aimed at understanding and deepening recent developments where Feynman and string amplitudes on the one hand, and periods and multiple zeta values on the other, have been at the heart of lively and fruitful interactions between theoretical physics and number theory over the past few decades. In this book, the reader will find research papers as well as survey articles, including open problems, on the interface between number theory, quantum field theory and string theory, written by leading experts in the respective fields. Topics include, among others, elliptic periods viewed from both a mathematical and a physical standpoint; further relations between periods and high energy physics, including cluster algebras and renormalisation theory; multiple Eisenstein series and q-analogues of multiple zeta values (also in connection with renormalisation); double shuffle and duality relations; alternative presentations of multiple zeta values using Ecalle's theory of moulds and arborification; a distribution formula for generalised complex and l-adic polylogarithms; Galois action on knots.

Given its scope, the book offers a valuable resource for researchers and graduate students interested in topics related to both quantum field theory, in particular, scattering amplitudes, and number theory.

Learning Modern Algebra aligns with the CBMS Mathematical Education of Teachers II recommendations, in both content and practice. It emphasizes rings and fields over groups, and it makes explicit connections between the ideas of abstract algebra and the mathematics used by high school teachers. It provides opportunities for prospective and practicing teachers to experience mathematics for themselves, before the formalities are developed, and it is explicit about the mathematical habits of mind that lie beneath the definitions and theorems. This book is designed for prospective and practicing high school mathematics teachers, but it can serve as a text for standard abstract algebra courses as well. The presentation is organized historically: the Babylonians introduced Pythagorean triples to teach the Pythagorean theorem; these were classified by Diophantus, and eventually this led Fermat to conjecture his Last Theorem. The text shows how much of modern algebra arose in attempts to prove this; it also shows how other important themes in algebra arose from questions related to teaching. Indeed, modern algebra is a very useful tool for teachers, with deep connections to the actual content of high school mathematics, as well as to the mathematics teachers use in their profession that doesn't necessarily ""end up on the blackboard."" The focus is on number theory, polynomials, and commutative rings. Group theory is introduced near the end of the text to explain why generalizations of the quadratic formula do not exist for polynomials of high degree, allowing the reader to appreciate the more general work of Galois and Abel on roots of polynomials. Results and proofs are motivated with specific examples whenever possible, so that abstractions emerge from concrete experience. Applications range from the theory of repeating decimals to the use of imaginary quadratic fields to construct problems with rational solutions. While such applications are integrated throughout, each chapter also contains a section giving explicit connections between the content of the chapter and high school teaching.

In the fall of 1990, I taught Math 581 at New Mexico State University for the first time. This course on field theory is the first semester of the year-long graduate algebra course here at NMSU. In the back of my mind, I thought it would be nice someday to write a book on field theory, one of my favorite mathematical subjects, and I wrote a crude form of lecture notes that semester. Those notes sat undisturbed for three years until late in 1993 when I finally made the decision to turn the notes into a book. The notes were greatly expanded and rewritten, and they were in a form sufficient to be used as the text for Math 581 when I taught it again in the fall of 1994. Part of my desire to write a textbook was due to the nonstandard format of our graduate algebra sequence. The first semester of our sequence is field theory. Our graduate students generally pick up group and ring theory in a senior-level course prior to taking field theory. Since we start with field theory, we would have to jump into the middle of most graduate algebra textbooks. This can make reading the text difficult by not knowing what the author did before the field theory chapters. Therefore, a book devoted to field theory is desirable for us as a text. While there are a number of field theory books around, most of these were less complete than I wanted.

A clear, efficient exposition of this topic with complete proofs and exercises, covering cubic and quartic formulas; fundamental theory of Galois theory; insolvability of the quintic; Galoiss Great Theorem; and computation of Galois groups of cubics and quartics. Suitable for first-year graduate students, either as a text for a course or for study outside the classroom, this new edition has been completely rewritten in an attempt to make proofs clearer by providing more details. It now begins with a short section on symmetry groups of polygons in the plane, for there is an analogy between polygons and their symmetry groups and polynomials and their Galois groups - an analogy which serves to help readers organise the various field theoretic definitions and constructions. The text is rounded off by appendices on group theory, ruler-compass constructions, and the early history of Galois Theory. The exposition has been redesigned so that the discussion of solvability by radicals now appears later and several new theorems not found in the first edition are included.

This text presents a careful introduction to methods of cryptology and error correction in wide use throughout the world and the concepts of abstract algebra and number theory that are essential for understanding these methods. The objective is to provide a thorough understanding of RSA, Diffie–Hellman, and Blum–Goldwasser cryptosystems and Hamming and Reed–Solomon error correction: how they are constructed, how they are made to work efficiently, and also how they can be attacked. To reach that level of understanding requires and motivates many ideas found in a first course in abstract algebra—rings, fields, finite abelian groups, basic theory of numbers, computational number theory, homomorphisms, ideals, and cosets. Those who complete this book will have gained a solid mathematical foundation for more specialized applied courses on cryptology or error correction, and should also be well prepared, both in concepts and in motivation, to pursue more advanced study in algebra and number theory. This text is suitable for classroom or online use or for independent study. Aimed at students in mathematics, computer science, and engineering, the prerequisite includes one or two years of a standard calculus sequence. Ideally the reader will also take a concurrent course in linear algebra or elementary matrix theory. A solutions manual for the 400 exercises in the book is available to instructors who adopt the text for their course.

It is by no means clear what comprises the "heart" or "core" of algebra, the part of algebra which every algebraist should know. Hence we feel that a book on "our heart" might be useful. We have tried to catch this heart in a collection of about 150 short sections, written by leading algebraists in these areas. These sections are organized in 9 chapters A, B, . . . , I. Of course, the selection is partly based on personal preferences, and we ask you for your understanding if some selections do not meet your taste (for unknown reasons, we only had problems in the chapter "Groups" to get enough articles in time). We hope that this book sets up a standard of what all algebraists are supposed to know in "their" chapters; interested people from other areas should be able to get a quick idea about the area. So the target group consists of anyone interested in algebra, from graduate students to established researchers, including those who want to obtain a quick overview or a better understanding of our selected topics. The prerequisites are something like the contents of standard textbooks on higher algebra. This book should also enable the reader to read the "big" Handbook (Hazewinkel 1999-) and other handbooks. In case of multiple authors, the authors are listed alphabetically; so their order has nothing to do with the amounts of their contributions.

Proceedings of the Conference on Algebra and Algebraic Geometry with Applications, July 19 – 26, 2000, at Purdue University to honor Professor Shreeram S. Abhyankar on the occasion of his seventieth birthday. Eighty-five of Professor Abhyankar's students, collaborators, and colleagues were invited participants. Sixty participants presented papers related to Professor Abhyankar's broad areas of mathematical interest. Sessions were held on algebraic geometry, singularities, group theory, Galois theory, combinatorics, Drinfield modules, affine geometry, and the Jacobian problem. This volume offers an outstanding collection of papers by expert authors.

Differential Galois theory has seen intense research activity during the last decades in several directions: elaboration of more general theories, computational aspects, model theoretic approaches, applications to classical and quantum mechanics as well as to other mathematical areas such as number theory. This book intends to introduce the reader to this subject by presenting Picard-Vessiot theory, i.e. Galois theory of linear differential equations, in a self-contained way. The needed prerequisites from algebraic geometry and algebraic groups are contained in the first two parts of the book. The third part includes Picard-Vessiot extensions, the fundamental theorem of Picard-Vessiot theory, solvability by quadratures, Fuchsian equations, monodromy group and Kovacic's algorithm. Over one hundred exercises will help to assimilate the concepts and to introduce the reader to some topics beyond the scope of this book. This book is suitable for a graduate course in differential Galois theory. The last chapter contains several suggestions for further reading encouraging the reader to enter more deeply into different topics of differential Galois theory or related fields.

This volume is based on talks given at the Workshop on Categorical Structures for Descent and Galois Theory, Hopf Algebras, and Semiabelian Categories held at The Fields Institute for Research in Mathematical Sciences (Toronto, ON, Canada). The meeting brought together researchers working in these interrelated areas. This collection of survey and research papers gives an up-to-date account of the many current connections among Galois theories, Hopf algebras, and semiabelian categories. The book features articles by leading researchers on a wide range of themes, specifically, abstract Galois theory, Hopf algebras, and categorical structures, in particular quantum categories and higher-dimensional structures. Articles are suitable for graduate students and researchers, specifically those interested in Galois theory and Hopf algebras and their categorical unification.

This graduate textbook offers an introduction to modern methods in number theory. It gives a complete account of the main results of class field theory as well as the Poitou-Tate duality theorems, considered crowning achievements of modern number theory. Assuming a first graduate course in algebra and number theory, the book begins with an introduction to group and Galois cohomology. Local fields and local class field theory, including Lubin-Tate formal group laws, are covered next, followed by global class field theory and the description of abelian extensions of global fields. The final part of the book gives an accessible yet complete exposition of the Poitou-Tate duality theorems. Two appendices cover the necessary background in homological algebra and the analytic theory of Dirichlet L-series, including the ?ebotarev density theorem. Based on several advanced courses given by the author, this textbook has been written for graduate students. Including complete proofs and numerous exercises, the book will also appeal to more experienced mathematicians, either as a text to learn the subject or as a reference.

Galois theory is the culmination of a centuries-long search for a solution to the classical problem of solving algebraic equations by radicals. In this book, Bewersdorff follows the historical development of the theory, emphasizing concrete examples along the way. As a result, many mathematical abstractions are now seen as the natural consequence of particular investigations. Few prerequisites are needed beyond general college mathematics, since the necessary ideas and properties of groups and fields are provided as needed. Results in Galois theory are formulated first in a concrete, elementary way, then in the modern form. Each chapter begins with a simple question that gives the reader an idea of the nature and difficulty of what lies ahead. The applications of the theory to geometric constructions, including the ancient problems of squaring the circle, duplicating the cube, and trisecting the angle, and the construction of regular n n-gons are also presented. This new edition contains an additional chapter as well as twenty facsimiles of milestones of classical algebra. It is suitable for undergraduates and graduate students, as well as teachers and mathematicians seeking a historical and stimulating perspective on the field.

The legacy of Galois was the beginning of Galois theory as well as group theory. From this common origin, the development of group theory took its own course, which led to great advances in the latter half of the 20th cen tury. It was John Thompson who shaped finite group theory like no-one else, leading the way towards a major milestone of 20th century mathematics, the classification of finite simple groups. After the classification was announced around 1980, it was again J. Thomp son who led the way in exploring its implications for Galois theory. The first question is whether all simple groups occur as Galois groups over the rationals (and related fields), and secondly, how can this be used to show that all finite groups occur (the 'Inverse Problem of Galois Theory'). What are the implica tions for the stmcture and representations of the absolute Galois group of the rationals (and other fields)? Various other applications to algebra and number theory have been found, most prominently, to the theory of algebraic curves (e.g., the Guralnick-Thompson Conjecture on the Galois theory of covers of the Riemann sphere).

The work of Joseph Fels Ritt and Ellis Kolchin in differential algebra paved the way for exciting new applications in constructive symbolic computation, differential Galois theory, the model theory of fields, and Diophantine geometry. This volume assembles Kolchin's mathematical papers, contributing solidly to the archive on construction of modern differential algebra. This collection of Kolchin's clear and comprehensive papers--in themselves constituting a history of the subject--is an invaluable aid to the student of differential algebra. In 1910, Ritt created a theory of algebraic differential equations modeled not on the existing transcendental methods of Lie, but rather on the new algebra being developed by E. Noether and B. van der Waerden. Building on Ritt's foundation, and deeply influenced by Weil and Chevalley, Kolchin opened up Ritt theory to modern algebraic geometry. In so doing, he led differential geometry in a new direction. By creating differential algebraic geometry and the theory of differential algebraic groups, Kolchin provided the foundation for a ``new geometry'' that has led to both a striking and an original approach to arithmetic algebraic geometry. Intriguing possibilities were introduced for a new language for nonlinear differential equations theory. The volume includes commentary by A. Borel, M. Singer, and B. Poizat. Also Buium and Cassidy trace the development of Kolchin's ideas, from his important early work on the differential Galois theory to his later groundbreaking results on the theory of differential algebraic geometry and differential algebraic groups. Commentaries are self-contained with numerous examples of various aspects of differential algebra and its applications. Central topics of Kolchin's work are discussed, presenting the history of differential algebra and exploring how his work grew from and transformed the work of Ritt. New directions of differential algebra are illustrated, outlining important current advances. Prerequisite to understanding the text is a background at the beginning graduate level in algebra, specifically commutative algebra, the theory of field extensions, and Galois theory.

Combining a concrete perspective with an exploration-based approach, Exploratory Galois Theory develops Galois theory at an entirely undergraduate level. The text grounds the presentation in the concept of algebraic numbers with complex approximations and assumes of its readers only a first course in abstract algebra. For readers with Maple or Mathematica, the text introduces tools for hands-on experimentation with finite extensions of the rational numbers, enabling a familiarity never before available to students of the subject. The text is appropriate for traditional lecture courses, for seminars, or for self-paced independent study by undergraduates and graduate students.

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The book gives a detailed account of the development of the theory of algebraic equations, from its origins in ancient times to its completion by Galois in the nineteenth century. The appropriate parts of works by Cardano, Lagrange, Vandermonde, Gauss, Abel, and Galois are reviewed and placed in their historical perspective, with the aim of conveying to the reader a sense of the way in which the theory of algebraic equations has evolved and has led to such basic mathematical notions as "group" and "field". A brief discussion of the fundamental theorems of modern Galois theory and complete proofs of the quoted results are provided, and the material is organized in such a way that the more technical details can be skipped by readers who are interested primarily in a broad survey of the theory. In this second edition, the exposition has been improved throughout and the chapter on Galois has been entirely rewritten to better reflect Galois' highly innovative contributions. The text now follows more closely Galois' memoir, resorting as sparsely as possible to anachronistic modern notions such as field extensions. The emerging picture is a surprisingly elementary approach to the solvability of equations by radicals, and yet is unexpectedly close to some of the most recent methods of Galois theory.

Insightful overview of many kinds of algebraic structures that are ubiquitous in mathematics. For researchers at graduate level and beyond.

This book is a collection of three introductory tutorials coming out of three courses given at the CIMPA Research School “Galois Theory of Difference Equations” in Santa Marta, Columbia, July 23–August 1, 2012. The aim of these tutorials is to introduce the reader to three Galois theories of linear difference equations and their interrelations. Each of the three articles addresses a different galoisian aspect of linear difference equations. The authors motivate and give elementary examples of the basic ideas and techniques, providing the reader with an entry to current research. In addition each article contains an extensive bibliography that includes recent papers; the authors have provided pointers to these articles allowing the interested reader to explore further.

This first volume develops factorization algebras with a focus upon examples exhibiting their use in field theory, which will be useful for researchers and graduates.

This book discusses major topics in Galois theory and advanced linear algebra, including canonical forms. Divided into four chapters and presenting numerous new theorems, it serves as an easy-to-understand textbook for undergraduate students of advanced linear algebra, and helps students understand other courses, such as Riemannian geometry. The book also discusses key topics including Cayley–Hamilton theorem, Galois groups, Sylvester’s law of inertia, Eisenstein criterion, and solvability by radicals. Readers are assumed to have a grasp of elementary properties of groups, rings, fields, and vector spaces, and familiarity with the elementary properties of positive integers, inner product space of finite dimension and linear transformations is beneficial.

This text aims to provide graduate students with a self-contained introduction to topics that are at the forefront of modern algebra, namely, coalgebras, bialgebras and Hopf algebras. The last chapter (Chapter 4) discusses several applications of Hopf algebras, some of which are further developed in the author’s 2011 publication, An Introduction to Hopf Algebras. The book may be used as the main text or as a supplementary text for a graduate algebra course. Prerequisites for this text include standard material on groups, rings, modules, algebraic extension fields, finite fields and linearly recursive sequences. The book consists of four chapters. Chapter 1 introduces algebras and coalgebras over a field K; Chapter 2 treats bialgebras; Chapter 3 discusses Hopf algebras and Chapter 4 consists of three applications of Hopf algebras. Each chapter begins with a short overview and ends with a collection of exercises which are designed to review and reinforce the material. Exercises range from straightforward applications of the theory to problems that are devised to challenge the reader. Questions for further study are provided after selected exercises. Most proofs are given in detail, though a few proofs are omitted since they are beyond the scope of this book.

Differential Galois theory studies solutions of differential equations over a differential base field. In much the same way that ordinary Galois theory is the theory of field extensions generated by solutions of (one variable) polynomial equations, differential Galois theory looks at the nature of the differential field extension generated by the solutions of differential equations. An additional feature is that the corresponding differential Galois groups (of automorphisms of the extension fixing the base and commuting with the derivation) are algebraic groups. This book deals with the differential Galois theory of linear homogeneous differential equations, whose differential Galois groups are algebraic matrix groups.In addition to providing a convenient path to Galois theory, this approach also leads to the constructive solution of the inverse problem of differential Galois theory for various classes of algebraic groups. Providing a self-contained development and many explicit examples, this book provides a unique approach to differential Galois theory and is suitable as a textbook at the advanced graduate level.

Collection of articles by leading experts in Galois theory, focusing on the Inverse Galois Problem.

This collection consists of original work on Galois theory, rings and algebras, algebraic geometry, group representations, algebraic K—theory and some of their applications.

Galois theory is considered one of the most beautiful subjects in mathematics, but it is hard to appreciate this fact fully without seeing specific examples. Numerous examples are therefore included throughout this text, in the hope that they will lead to a deeper understanding and genuine appreciation of the more abstract and advanced literature on Galois theory.

Suitable for advanced undergraduates and graduate students in mathematics and computer science, this precise, self-contained treatment of Galois theory features detailed proofs and complete solutions to exercises. Originally published in French as Algèbre — Polynômes, théorie de Galois et applications informatiques, this 2017 Dover Aurora edition marks the volume's first English-language publication. The three-part treatment begins by providing the essential introduction to Galois theory. The second part is devoted to the algebraic, normal, and separable Galois extensions that constitute the center of the theory and examines abelian, cyclic, cyclotomic, and radical extensions. This section enables readers to acquire a comprehensive understanding of the Galois group of a polynomial. The third part deals with applications of Galois theory, including excellent discussions of several important real-world applications of these ideas, including cryptography and error-control coding theory. Symbolic computation via the Maple computer algebra system is incorporated throughout the text (though other software of symbolic computation could be used as well), along with a large number of very interesting exercises with full solutions.

Hopf algebras have been shown to play a natural role in studying questions of integral module structure in extensions of local or global fields. This book surveys the state of the art in Hopf-Galois theory and Hopf-Galois module theory and can be viewed as a sequel to the first author's book, Taming Wild Extensions: Hopf Algebras and Local Galois Module Theory, which was published in 2000. The book is divided into two parts. Part I is more algebraic and focuses on Hopf-Galois structures on Galois field extensions, as well as the connection between this topic and the theory of skew braces. Part II is more number theoretical and studies the application of Hopf algebras to questions of integral module structure in extensions of local or global fields. Graduate students and researchers with a general background in graduate-level algebra, algebraic number theory, and some familiarity with Hopf algebras will appreciate the overview of the current state of this exciting area and the suggestions for numerous avenues for further research and investigation.